GLOBAL FIT OF $b \to s\,\ell^+\ell^-$ AND $B \to K^*(\to K\pi)\,\ell^+\ell^-$ AT HIGH-q^2

based on arXiv:1006.5013 + 1105.0376 + 1111.2558

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LHCb-Theory Workshop

OUTLINE

1) Global fit of $b \to s\,\ell^+\ell^-$

   A) Experimental input

   B) Fit results

2) $B \to K^*(\to K\pi)\,\ell^+\ell^-$ at high-q^2

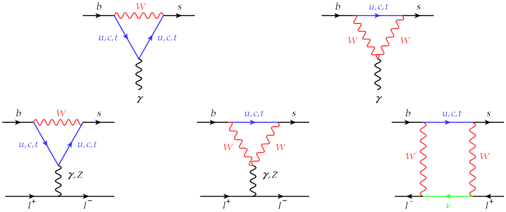
   A) Angular distribution and Observables

   B) NP sensitivity

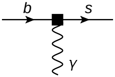
Global fit of $b \to s\,\ell^+\ell^-$

FCNC $b \to s + \{\gamma, \bar{\ell}\ell\}$ IN THE SM

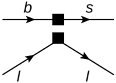
Main contributions within $\Delta B = 1$ EFT:



$\rightarrow \quad C_7^{\gamma} \quad \times$



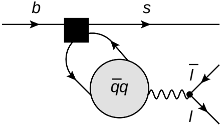
$\rightarrow \quad C_{9,10}^{\ell\ell} \quad \times$



Background contribution $B \to K^{(*)}(\bar{q}q) \to K^{(*)}\bar{\ell}\ell$

from 4-quark operators $b \to s\bar{q}q$

q^2 = dilepton invariant mass:



q^2 - REGIONS IN $b \to s + \bar{\ell}\ell$ $\quad$ $K^{(*)}$-ENERGY IN B-REST FRAME: $E_{K^{(*)}} = (M_B^2 + M_{K^{(*)}}^2 - q^2)/(2\,M_B)$

q^2-region	low-q^2: $q^2 \ll M_B^2$	high-q^2: $q^2 \sim M_B^2$
$K^{(*)}$-recoil	large recoil: $E_{K^{(*)}} \sim M_B/2$	low recoil: $E_{K^{(*)}} \sim M_{K^{(*)}} + \Lambda_{\rm QCD}$
theory method	QCDF, SCET: $q^2 \in [1, 6]$ GeV^2	OPE + HQET: $q^2 \geqslant (14 \ldots 15)$ GeV^2

$b \to s \ell^+ \ell^-$ DATA – NUMBER OF EVENTS

# of evts	BaBar 2008 384 M $\bar{B}B$	Belle 2009 605 fb^{-1}	CDF 2011 6.8 fb^{-1}	LHCb 2011 309 pb^{-1}
$B^0 \to K^{*0}\,\bar{\ell}\ell$	64 ± 16	$247 \pm 54^{\dagger}$	164 ± 15	323 ± 21
$B^+ \to K^{*+}\,\bar{\ell}\ell$			20 ± 6	
$B^+ \to K^+\,\bar{\ell}\ell$	53 ± 12	$162 \pm 38^{\dagger}$	234 ± 19	
$B^0 \to K^0_S\,\bar{\ell}\ell$			28 ± 9	
$B_s \to \phi\,\bar{\ell}\ell$			49 ± 7	
$\Lambda_b \to \Lambda\,\bar{\ell}\ell$			24 ± 5	

- CP-averaged results
- vetoed q^2 region around J/ψ and ψ' regions
- † unknown mixture of B^0 and $B^{\pm}$

Babar arXiv:0804.4412
Belle arXiv:0904.0770
CDF arXiv:1107.3753 + 1108.0695
LHCb LHCB-CONF-2011-038

Outlook/Prospects:

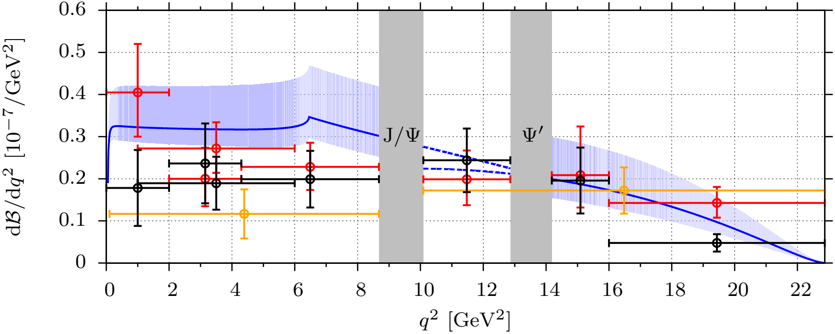
- Belle reprocessed all data 711 fb^{-1}
- CDF recorded perhaps about 10 fb^{-1} ???
- LHCb recorded now about 1 fb^{-1} $\to$ naively: about 1000 events
- SuperB expects about 10000-15000 $B \to K^*\bar{\ell}\ell$ events [A.J.Bevan arXiv:1110.3901]

$B \to K \ell^+ \ell^-$ DATA USED IN FIT

Data available in 6 q^2-bins for: $\langle Br \rangle$

in fit used: $[q^2_{min}, q^2_{max}] = [1.0, 6.0], [14.18, 16.0], [16.0, 22.86]$ GeV^2

[BaBar] [Belle] [CDF]



[SM predicition: CB/GH/DvD/Wacker arXiv:1111.2558]

$B \to K^* \ell^+ \ell^-$ DATA USED IN FIT

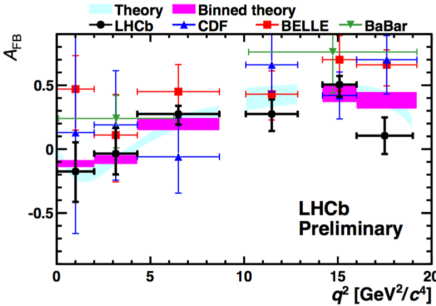
data in q^2-bins for: $\langle Br \rangle$, $\langle A_{FB} \rangle$, $\langle F_L \rangle$

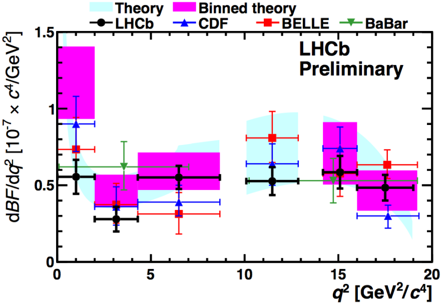
$[1.0, 6.0]$, $[14.18, 16.0]$, $[16.0, 19.2]$ GeV^2

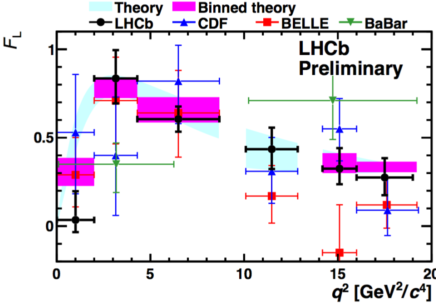
angular analysis in θ_ℓ and θ_{K*}: each q^2-bin

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_{K*}} = \frac{3}{2} F_L \cos^2\theta_{K*} + \frac{3}{4}(1 - F_L)\sin^2\theta_{K*},$$

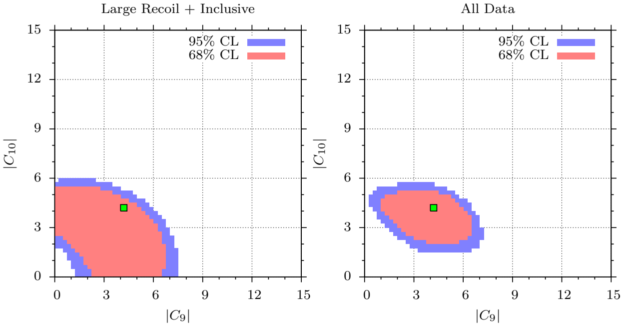
$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta_\ell} = \frac{3}{4} F_L \sin^2\theta_\ell + \frac{3}{8}(1 - F_L)(1 + \cos^2\theta_\ell) + A_{FB}\cos\theta_\ell$$







GLOBAL FIT OF C_9 AND C_{10} – COMPLEX



[CB/GH/DvD arXiv:1105.0376v4]

Scan resolution

$|C_7| \in [.30, .35], \qquad \Delta|C_7| = .01$

$|C_{9,10}| \in [0, 15], \qquad \Delta|C_{9,10}| = 0.25$

$\phi_7 \in [0, 2\pi), \qquad \Delta\phi_7 = \pi/16$

$\phi_{9,10} \in [0, 2\pi), \qquad \Delta\phi_{9,10} = \pi/16$

SM = green square

- $B \to X_s \bar{\ell}\ell$: Br in $q^2 \in [1, 6]$ GeV^2 [Babar/Belle]
- $B \to K^* \bar{\ell}\ell$: Br, $A_{\rm FB}$, F_L in $q^2 \in [1, 6]$ GeV^2

  [Belle/CDF] Br, $A_{\rm FB}$ in $q^2 \in [14.2, 16] + [> 16]$ GeV^2

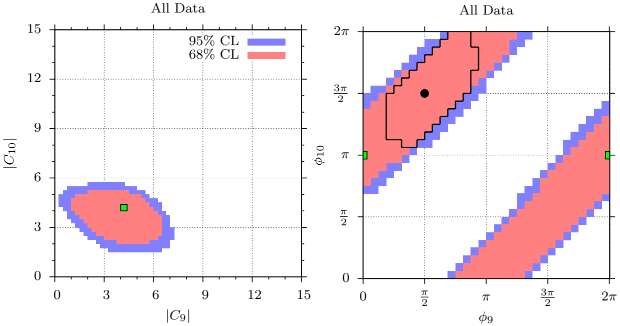
Before Summer 2011

Determining 68 (95) % CL in 6D pmr-space $|C_{7,9,10}|$ and $\phi_{7,9,10}$ → projection on $|C_9| - |C_{10}|$

⇒ without high-q^2 data [left] and with [right] → important impact,

BUT form factors from lattice very desirable !!!

Global Fit of C_9 and C_{10} – complex



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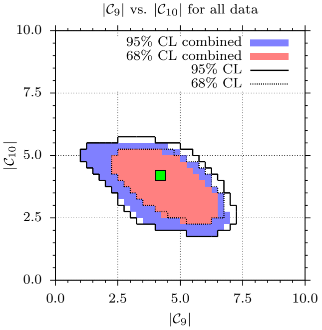

@ 95 % CL

$$1.0 \leqslant |C_9| \leqslant 7.0$$

$$1.8 \leqslant |C_{10}| \leqslant 5.5$$

$$Br(B_s \to \mu\bar{\mu}) \lesssim 1.7 \times Br_{\rm SM}$$

[CB/GH/DvD/Wacker arXiv:1111.2558v2]

Scan resolution

$|C_7| \in [.30, .40], \quad \Delta|C_7| = .02$

$|C_{9,10}| \in [0, 15], \quad \Delta|C_{9,10}| = 0.25$

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- $B \to K^*\bar{\ell}\ell$: Br, $A_{\rm FB}$, F_L in $q^2 \in [1, 6]$ GeV^2

  [Belle/CDF/LHCb] Br, $A_{\rm FB}$ in $q^2 \in [14.2, 16] + [> 16]$ GeV^2

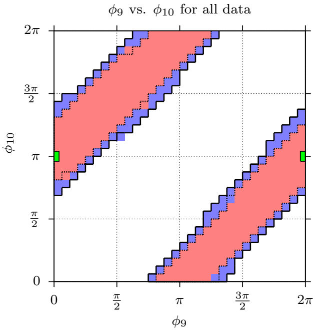
November 2011

Adding also . . .

- $B \to K\bar{\ell}\ell$: Br in $q^2 \in [1, 6]$ GeV^2

  [Belle/CDF] Br in $q^2 \in [14.2, 16] + [> 16]$ GeV^2

Global Fit of C_9 and C_{10} – complex



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November 2011

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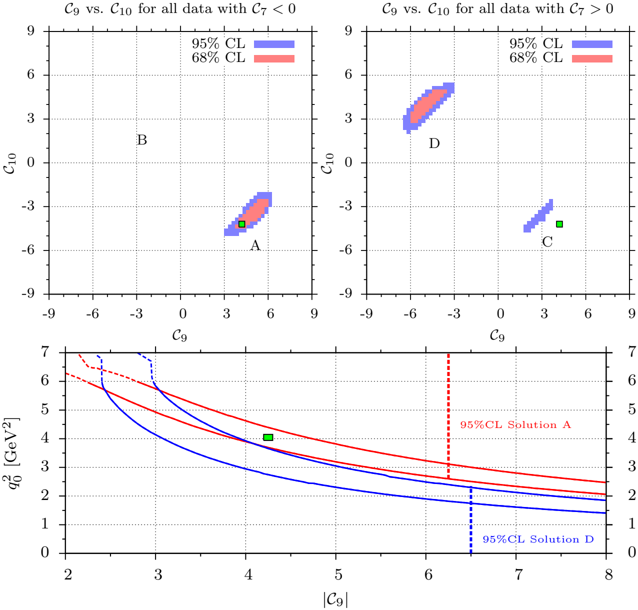
Global Fit of C_9 and C_{10} – real



$C_7^{\mathrm{SM}} < 0$

solution A:

$$3.0 < C_9 < 6.25$$
$$-5.0 < C_{10} < -2.0$$

solution D:

$$-6.5 < C_9 < -3.0$$
$$2.0 < C_{10} < 5.5$$

$\Rightarrow$ zero crossing of A_{FB}

solution A:

$$q_0^2 > 2.6\ \mathrm{GeV}^2$$

solution D:

$$q_0^2 > 1.7\ \mathrm{GeV}^2$$

OTHER MODEL-INDEPENDENT GLOBAL FITS

Descotes-Genon/Ghosh/Matias/Ramon arXiv:1104.3342

included data (before summer)

- $B \to X_s \gamma$ (Br),
  $B \to K^* \gamma$ (S, A_I)
- $B \to X_s \bar{\ell}\ell$ (low-q^2: Br),
  $B \to K^* \bar{\ell}\ell$ (only low-q^2: $A_{\rm FB}$, F_L)
- upper bound on $Br(B_s \to \bar{\mu}\mu)$

and NP in real Wilson coefficients

- $C_{7,7'}$
- + $C_{9,10}$
- + $C_{9',10'}$

Altmannshofer/Paradisi/Straub arXiv:1111.1257

included data (up to date)

- $B \to X_s \gamma$ (Br),
  $B \to K^* \gamma$ (S)
- $B \to X_s \bar{\ell}\ell$ (low + high-q^2: Br),
  $B \to K^* \bar{\ell}\ell$ (low + high-q^2: Br, $A_{\rm FB}$, F_L)

and NP in real and complex Wilson coefficients

- $C_{7,7',9,9',10,10'}$ (in varying stages)
- Z-penguin + $C_{7,7'}$
  $\Rightarrow$ relates $b \to s\,\bar{\ell}\ell$ and $b \to s\,\bar{\nu}\nu$

EOS Implementation

Dependencies

- written in C++0x, needs `>=g++-4.4`
- written for Linux, but any UNIXoid OS should do
- minimal library dependencies
- GNU Scientific Library for special functions, random number generation, simplex method
- HDF5 for input/output

Extent

- multi-threaded calculations (POSIX threads!)
- extensive collection of test cases
- $\sim$150 File of Code, $\sim$30k Lines of Code

http://project.het.physik.tu-dortmund.de/eos

$$B \to K^*(\to K\pi)\,\ell^+\ell^-$$

at high-q^2

KINEMATICS $B \to V[\to P_1 P_2] + \bar{\ell}\ell$

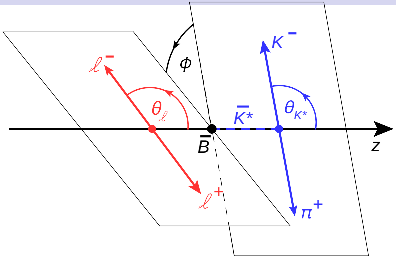
4-body decay with on-shell intermediate V(ector)

1) $q^2 = m^2_{\bar{\ell}\ell} = (p_{\bar{\ell}} + p_\ell)^2 = (p_B - p_V)^2$

2) $\cos\theta_\ell$ with $\theta_\ell \angle(\vec{p}_B, \vec{p}_{\bar{\ell}})$ in $(\bar{\ell}\ell)$ – c.m. system

3) $\cos\theta_V$ with $\theta_V \angle(\vec{p}_B, \vec{p}_1)$ in $(P_1 P_2)$ – c.m. system

4) $\phi \angle(\vec{p}_1 \times \vec{p}_2, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B-RF



$\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+) + \bar{\ell}\ell$: $\quad I_i^{(s,c)}(q^2)$ = "ANGULAR OBSERVABLES"

$$\frac{32\pi}{9}\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_{K*}\, d\phi} = I_1^s \sin^2\theta_{K*} + I_1^c \cos^2\theta_{K*} + (I_2^s \sin^2\theta_{K*} + I_2^c \cos^2\theta_{K*})\cos 2\theta_\ell$$
$$+ I_3 \sin^2\theta_{K*} \sin^2\theta_\ell \cos 2\phi + I_4 \sin 2\theta_{K*} \sin 2\theta_\ell \cos\phi + I_5 \sin 2\theta_{K*} \sin\theta_\ell \cos\phi$$
$$+ (I_6^s \sin^2\theta_{K*} + I_6^c \cos^2\theta_{K*})\cos\theta_\ell + I_7 \sin 2\theta_{K*} \sin\theta_\ell \sin\phi$$
$$+ I_8 \sin 2\theta_{K*} \sin 2\theta_\ell \sin\phi + I_9 \sin^2\theta_{K*} \sin^2\theta_\ell \sin 2\phi$$

$\Rightarrow$ "$2 \times (12 + 12) = 48$" if measured separately: A) decay + CP-conj & B) for $\ell = e, \mu$

$\Rightarrow$ for (SM + χ-flipped) operators and $m_\ell = 0$: $\quad I_1^s = 3 I_2^s, \quad I_1^c = -I_2^c, \quad I_6^c = 0, \quad$ +4th rel.

[Egede/Hurth/Matias/Ramon/Reece arXiv:1005.0571]

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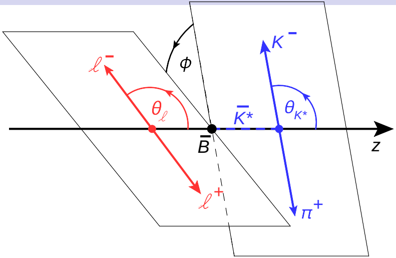

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$$+ (I_6^s \sin^2\theta_{K*} + I_6^c \cos^2\theta_{K*}) \cos\theta_\ell + I_7 \sin 2\theta_{K*} \sin\theta_\ell \sin\phi$$
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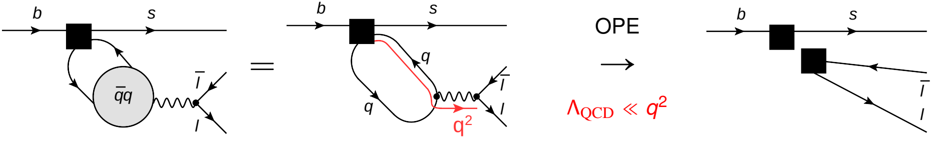
HIGH-q^2: OPE – I

Hard momentum transfer $(q^2 \sim M_B^2)$ through $(\bar{q}q) \to \bar{\ell}\ell$ allows local OPE



$$\mathcal{M}[\bar{B} \to \bar{K}^* + \bar{\ell}\ell] \sim \frac{8\pi^2}{q^2} i \int d^4x\, e^{iq\cdot x} \langle \bar{K}^* | T\{\mathcal{L}^{\rm eff}(0), j_\mu^{\rm em}(x)\} | \bar{B}\rangle\, [\bar{\ell}\gamma^\mu \ell]$$

$$= \left(\sum_a \mathcal{C}_{3a} \mathcal{Q}_{3a}^\mu + \sum_b \mathcal{C}_{5b} \mathcal{Q}_{5b}^\mu + \sum_c \mathcal{C}_{6c} \mathcal{Q}_{6c}^\mu + \mathcal{O}(dim > 6) \right) [\bar{\ell}\gamma_\mu \ell]$$

Buchalla/Isidori hep-ph/9801456, Grinstein/Pirjol hep-ph/0404250, Beylich/Buchalla/Feldmann arXiv:1101.5118

Leading $dim = 3$ operators: $\langle \bar{K}^* | \mathcal{Q}_{3,a} | \bar{B}\rangle \sim$ usual $B \to K^*$ form factors $V, A_{0,1,2}, T_{1,2,3}$

$$\mathcal{Q}_{3,1}^\mu = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) [\bar{s}\, \gamma_\nu (1 - \gamma_5)\, b] \quad \to \quad C_9 \to C_9^{\rm eff}, \quad (V, A_{0,1,2})$$

$$\mathcal{Q}_{3,2}^\mu = \frac{i m_b}{q^2} q_\nu\, [\bar{s}\, \sigma_{\nu\mu} (1 + \gamma_5)\, b] \quad \to \quad C_7 \to C_7^{\rm eff}, \quad (T_{1,2,3})$$

HIGH-q^2: OPE – II

$dim = 3$	α_s matching corrections are also known
$m_s \neq 0$	2 additional $dim = 3$ operators, suppressed with $\alpha_s m_s / m_b \sim 0.5$ %, NO new form factors
$dim = 4$	absent
$dim = 5$	suppressed by $(\Lambda_{\rm QCD}/m_b)^2 \sim 2$ %, explicite estimate @ $q^2 = 15$ GeV^2: $< 1\%$ [Beylich/Buchalla/Feldmann arXiv:1101.5118]
$dim = 6$	suppressed by $(\Lambda_{\rm QCD}/m_b)^3 \sim 0.2$ % and small QCD-penguin's: $C_{3,4,5,6}$ spectator quark effects: from weak annihilation
BEYOND OPE	duality violating effects [Beylich/Buchalla/Feldmann arXiv:1101.5118]

- based on Shifman model for *c*-quark correlator + fit to recent BES data
- ± 2 % for integrated rate $q^2 > 15$ GeV^2

$\Rightarrow$ OPE of exclusive $\bar{B} \to \bar{K}^*(\bar{K}) + \bar{\ell}\ell$ predicts small sub-leading contributions !!!

BUT, still missing $B \to K^*$ form factors @ high-q^2
for predictions of angular observables $I_i^{(k)}$

HIGH-q^2 – TRANSVERSITY AMPLITUDES

$$A_\perp^{L,R} = +\left[C^{L,R} + \tilde{r}_a\right] f_\perp\,, \qquad A_\parallel^{L,R} = -\left[C^{L,R} + \tilde{r}_b\right] f_\parallel\,,$$

$$A_0^{L,R} = -C^{L,R} f_0 - N M_B \frac{(1-\hat{s}-\hat{M}_{K*}^2)(1+\hat{M}_{K*})^2 \tilde{r}_b A_1 - \hat{\lambda}\,\tilde{r}_c A_2}{2\,\hat{M}_{K*}(1+\hat{M}_{K*})\sqrt{\hat{s}}}$$

$\Rightarrow$ Universal short-distance coefficients: $C^{L,R} = C_9^{\text{eff}} + \kappa \frac{2 m_b M_B}{q^2} C_7^{\text{eff}} \mp C_{10}$
(SM: $C_9 \sim +4$, $C_{10} \sim -4$, $C_7 \sim -0.3$)

known structure of sub-leading corrections [Grinstein/Pirjol hep-ph/0404250]

$$\tilde{r}_i \sim \pm \frac{\Lambda_{\text{QCD}}}{m_b} \left(C_7^{\text{eff}} + \alpha_s(\mu) e^{i\delta_i}\right)\,, \qquad i = a, b, c$$

form factors ("helicity FF's" [Bharucha/Feldmann/Wick arXiv:1004.3249])

$$f_\perp = \frac{\sqrt{2\hat{\lambda}}}{1+\hat{M}_{K*}} V, \quad f_\parallel = \sqrt{2}\,(1+\hat{M}_{K*})\,A_1, \quad f_0 = \frac{(1-\hat{s}-\hat{M}_{K*}^2)(1+\hat{M}_{K*})^2 A_1 - \hat{\lambda}\,A_2}{2\,\hat{M}_{K*}(1+\hat{M}_{K*})\sqrt{\hat{s}}}$$

High-q^2 – SM operator basis

$$(2\,I_2^s + I_3) = 2\,\rho_1 \times f_\perp^2, \qquad -I_2^c = 2\,\rho_1 \times f_0^2, \qquad I_5/\sqrt{2} = 4\,\rho_2 \times f_0 f_\perp,$$

$$(2\,I_2^s - I_3) = 2\,\rho_1 \times f_\parallel^2, \qquad \sqrt{2}\,I_4 = 2\,\rho_1 \times f_0 f_\parallel, \qquad I_6^s/2 = 4\,\rho_2 \times f_\parallel f_\perp,$$

$$I_7 = I_8 = I_9 = 0, \qquad (I_6^c = 0) \qquad (m_\ell = 0)$$

A) ρ_1 and ρ_2 are largely μ-scale independent and B) $f_{\perp,\parallel,0}$ FF-dependent

$$\rho_1(q^2) \equiv \left| C_9^{\rm eff} + \kappa \frac{2m_b^2}{q^2} C_7^{\rm eff} \right|^2 + |C_{10}|^2, \qquad \rho_2(q^2) \equiv {\rm Re}\left(C_9^{\rm eff} + \kappa \frac{2m_b^2}{q^2} C_7^{\rm eff} \right) C_{10}^*$$

$$\frac{d\Gamma}{dq^2} = 2\,\rho_1 \times (f_0^2 + f_\perp^2 + f_\parallel^2), \qquad A_{\rm FB} = 3\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp f_\parallel}{(f_0^2 + f_\perp^2 + f_\parallel^2)},$$

$$F_{\rm L} = \frac{f_0^2}{f_0^2 + f_\perp^2 + f_\parallel^2}, \qquad A_T^{(2)} = \frac{f_\perp^2 - f_\parallel^2}{f_\perp^2 + f_\parallel^2}, \qquad A_T^{(3)} = \frac{f_\parallel}{f_\perp}, \qquad A_T^{(4)} = 2\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp}{f_\parallel}$$

Short-distance-free ratios !!! TEST lattice vs exp. data + OPE or FIT FF-shapes !!!

$$\frac{f_0}{f_\parallel} = \frac{\sqrt{2} I_5}{I_6} = \frac{-I_2^c}{\sqrt{2} I_4} = \frac{\sqrt{2} I_4}{2 I_2^s - I_3} = \sqrt{\frac{-I_2^c}{2 I_2^s - I_3}},$$

$$\frac{f_\perp}{f_\parallel} = \sqrt{\frac{2 I_2^s + I_3}{2 I_2^s - I_3}} = \frac{\sqrt{-I_2^c\,(2 I_2^s + I_3)}}{\sqrt{2} I_4}, \qquad \frac{f_0}{f_\perp} = \sqrt{\frac{-I_2^c}{2 I_2^s + I_3}}$$

High-q^2 – SM operator basis

$$(2\,I_2^s + I_3) = 2\,\rho_1 \times f_\perp^2, \qquad -I_2^c = 2\,\rho_1 \times f_0^2, \qquad I_5/\sqrt{2} = 4\,\rho_2 \times f_0 f_\perp,$$

$$(2\,I_2^s - I_3) = 2\,\rho_1 \times f_\parallel^2, \qquad \sqrt{2}\,I_4 = 2\,\rho_1 \times f_0 f_\parallel, \qquad I_6^s/2 = 4\,\rho_2 \times f_\parallel f_\perp,$$

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$$\frac{d\Gamma}{dq^2} = 2\,\rho_1 \times (f_0^2 + f_\perp^2 + f_\parallel^2), \qquad A_{\rm FB} = 3\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp f_\parallel}{(f_0^2 + f_\perp^2 + f_\parallel^2)},$$

$$F_{\rm L} = \frac{f_0^2}{f_0^2 + f_\perp^2 + f_\parallel^2}, \qquad A_T^{(2)} = \frac{f_\perp^2 - f_\parallel^2}{f_\perp^2 + f_\parallel^2}, \qquad A_T^{(3)} = \frac{f_\parallel}{f_\perp}, \qquad A_T^{(4)} = 2\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp}{f_\parallel}$$

Short-distance-free ratios !!! TEST lattice vs exp. data + OPE or FIT FF-shapes !!!

$$\frac{f_0}{f_\parallel} = \frac{\sqrt{2} I_5}{I_6} = \frac{-I_2^c}{\sqrt{2} I_4} = \frac{\sqrt{2} I_4}{2 I_2^s - I_3} = \sqrt{\frac{-I_2^c}{2 I_2^s - I_3}},$$

$$\frac{f_\perp}{f_\parallel} = \sqrt{\frac{2 I_2^s + I_3}{2 I_2^s - I_3}} = \frac{\sqrt{-I_2^c\,(2 I_2^s + I_3)}}{\sqrt{2} I_4}, \qquad \frac{f_0}{f_\perp} = \sqrt{\frac{-I_2^c}{2 I_2^s + I_3}}$$

HIGH-q^2 – SM OPERATOR BASIS

$$(2\,I_2^s + I_3) = 2\,\rho_1 \times f_\perp^2, \qquad -I_2^c = 2\,\rho_1 \times f_0^2, \qquad I_5/\sqrt{2} = 4\,\rho_2 \times f_0 f_\perp,$$

$$(2\,I_2^s - I_3) = 2\,\rho_1 \times f_\parallel^2, \qquad \sqrt{2}\,I_4 = 2\,\rho_1 \times f_0 f_\parallel, \qquad I_6^s/2 = 4\,\rho_2 \times f_\parallel f_\perp,$$

$$I_7 = I_8 = I_9 = 0, \qquad (I_6^c = 0) \qquad (m_\ell = 0)$$

A) ρ_1 and ρ_2 are largely μ-scale independent and B) $f_{\perp,\parallel,0}$ FF-dependent

$$\rho_1(q^2) \equiv \left| C_9^{\rm eff} + \kappa \frac{2m_b^2}{q^2} C_7^{\rm eff} \right|^2 + |C_{10}|^2, \qquad \rho_2(q^2) \equiv {\rm Re}\left(C_9^{\rm eff} + \kappa \frac{2m_b^2}{q^2} C_7^{\rm eff} \right) C_{10}^*$$

$$\frac{d\Gamma}{dq^2} = 2\,\rho_1 \times (f_0^2 + f_\perp^2 + f_\parallel^2), \qquad A_{\rm FB} = 3\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp f_\parallel}{(f_0^2 + f_\perp^2 + f_\parallel^2)},$$

$$F_{\rm L} = \frac{f_0^2}{f_0^2 + f_\perp^2 + f_\parallel^2}, \qquad A_T^{(2)} = \frac{f_\perp^2 - f_\parallel^2}{f_\perp^2 + f_\parallel^2}, \qquad A_T^{(3)} = \frac{f_\parallel}{f_\perp}, \qquad A_T^{(4)} = 2\,\frac{\rho_2}{\rho_1} \times \frac{f_\perp}{f_\parallel}$$

Short-distance-free ratios !!! TEST lattice vs exp. data + OPE or FIT FF-shapes !!!

$$\frac{f_0}{f_\parallel} = \frac{\sqrt{2} I_5}{I_6} = \frac{-I_2^c}{\sqrt{2} I_4} = \frac{\sqrt{2} I_4}{2 I_2^s - I_3} = \sqrt{\frac{-I_2^c}{2 I_2^s - I_3}},$$

$$\frac{f_\perp}{f_\parallel} = \sqrt{\frac{2 I_2^s + I_3}{2 I_2^s - I_3}} = \frac{\sqrt{-I_2^c\,(2 I_2^s + I_3)}}{\sqrt{2} I_4}, \qquad \frac{f_0}{f_\perp} = \sqrt{\frac{-I_2^c}{2 I_2^s + I_3}}$$

HIGH-q^2 – "LONG-DISTANCE FREE"

FF-FREE RATIOS !!! TEST SD FLAVOUR COUPLINGS VERSUS EXP. DATA + OPE

$$H_T^{(1)} = \frac{\sqrt{2} I_4}{\sqrt{-I_2^c \left(2 I_2^s - I_3\right)}} = sgn(f_0) \cdot 1$$

$$H_T^{(2)} = \frac{I_5}{\sqrt{-2 I_2^c \left(2 I_2^s + I_3\right)}} = 2\,\frac{\rho_2}{\rho_1}, \qquad H_T^{(3)} = \frac{I_6}{2\sqrt{(2 I_2^s)^2 - I_3^2}} = 2\,\frac{\rho_2}{\rho_1}$$

FF-FREE CP-ASYMMETRIES: SM OPERATOR BASIS [CB/GH/DvD arXiv:1105.0367]

$$a_{\rm CP}^{(1)} = \frac{\rho_1 - \bar\rho_1}{\rho_1 + \bar\rho_1}, \qquad a_{\rm CP}^{(2)} = \frac{\frac{\rho_2}{\rho_1} - \frac{\bar\rho_2}{\bar\rho_1}}{\frac{\rho_2}{\rho_1} + \frac{\bar\rho_2}{\bar\rho_1}}, \qquad a_{\rm CP}^{(3)} = 2\,\frac{\rho_2 - \bar\rho_2}{\rho_1 + \bar\rho_1}$$

- NLO QCD corrections large $\Rightarrow$ decrease CP-asymmetries
- still, theoretical uncertainties large: dominated by renorm. scale μ_b
- time-integrated $a_{\rm CP}^{mix}$ in $B_s \to \phi(\to K^+K^-) + \bar\ell\ell$ is CP-odd = untagged
- @ high-q^2: $A_{\rm CP}[B \to K\bar\ell\ell] = a_{\rm CP}^{(1)}[B \to K^*\bar\ell\ell]$

High-q^2 – "long-distance free"

FF-free ratios — !!! test SD flavour couplings versus exp. data + OPE

$$H_T^{(1)} = \frac{\sqrt{2} I_4}{\sqrt{-I_2^c \left(2 I_2^s - I_3\right)}} = sgn(f_0) \cdot 1$$

$$H_T^{(2)} = \frac{I_5}{\sqrt{-2 I_2^c \left(2 I_2^s + I_3\right)}} = 2\,\frac{\rho_2}{\rho_1}, \qquad H_T^{(3)} = \frac{I_6}{2\sqrt{(2 I_2^s)^2 - I_3^2}} = 2\,\frac{\rho_2}{\rho_1}$$

FF-free CP-asymmetries: SM operator basis — [CB/GH/DvD arXiv:1105.0367]

$$a_{\rm CP}^{(1)} = \frac{\rho_1 - \bar\rho_1}{\rho_1 + \bar\rho_1}, \qquad a_{\rm CP}^{(2)} = \frac{\frac{\rho_2}{\rho_1} - \frac{\bar\rho_2}{\bar\rho_1}}{\frac{\rho_2}{\rho_1} + \frac{\bar\rho_2}{\bar\rho_1}}, \qquad a_{\rm CP}^{(3)} = 2\,\frac{\rho_2 - \bar\rho_2}{\rho_1 + \bar\rho_1}$$

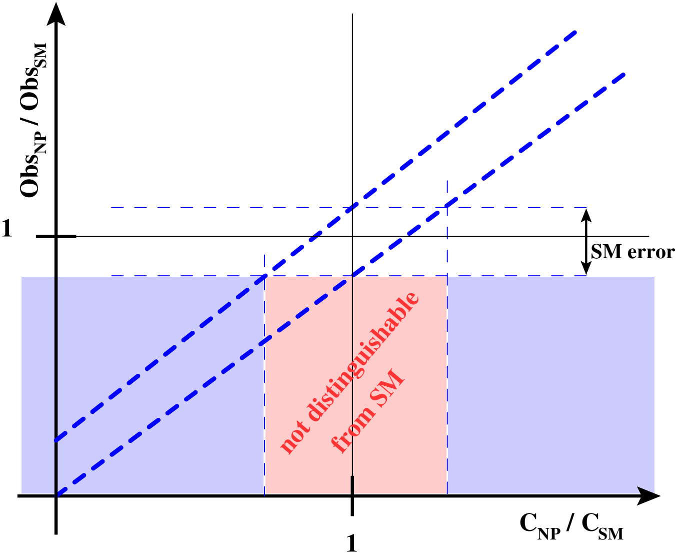
- NLO QCD corrections large $\Rightarrow$ decrease CP-asymmetries
- still, theoretical uncertainties large: dominated by renorm. scale μ_b
- time-integrated $a_{\rm CP}^{mix}$ in $B_s \to \phi(\to K^+ K^-) + \bar\ell\ell$ is CP-odd = untagged
- @ high-q^2: $A_{\rm CP}[B \to K\bar\ell\ell] = a_{\rm CP}^{(1)}[B \to K^*\bar\ell\ell]$

COMMENTS

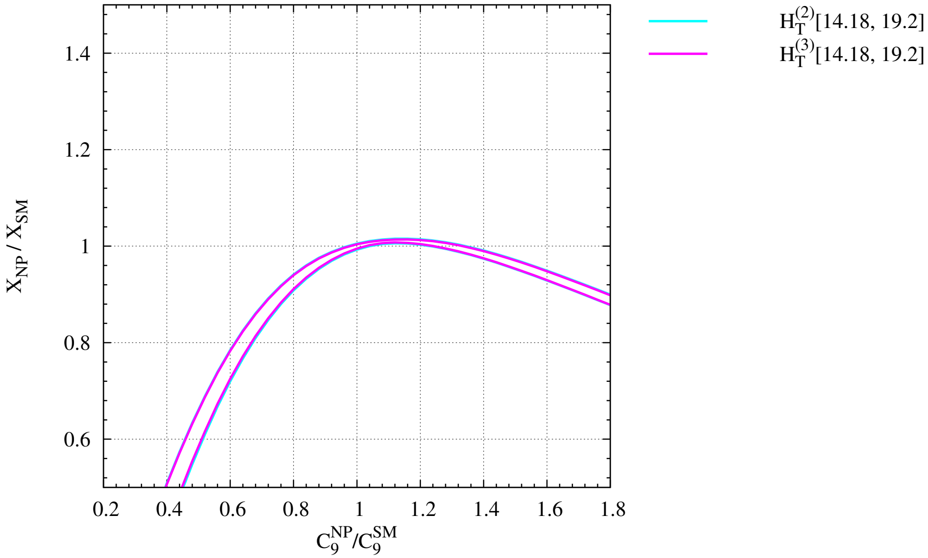
For SM operator basis($C_{7,9,10}$):

- measure all angular observables $I_i^{(s,c)}$
- test $H_T^{(1)} = 1$ and $H_T^{(2)} = H_T^{(3)}$ → deviations signal problem with OPE
- $H_T^{(2,3)}$ are better than A_{FB} (or $A_T^{(4)}$) @Êhigh-q^2
- short-distance-free ratios → give handle on $B \to K^*$ form factor ratios

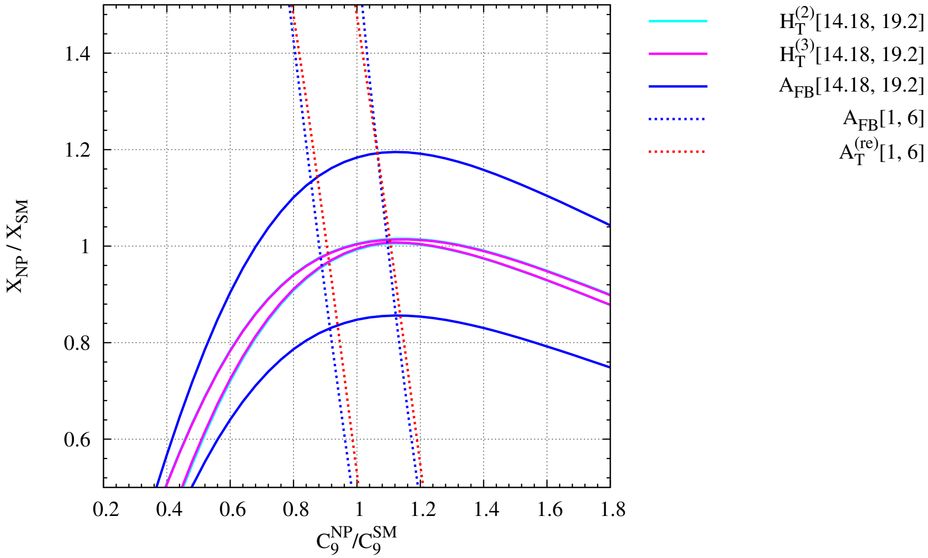
NP SENSITIVITY



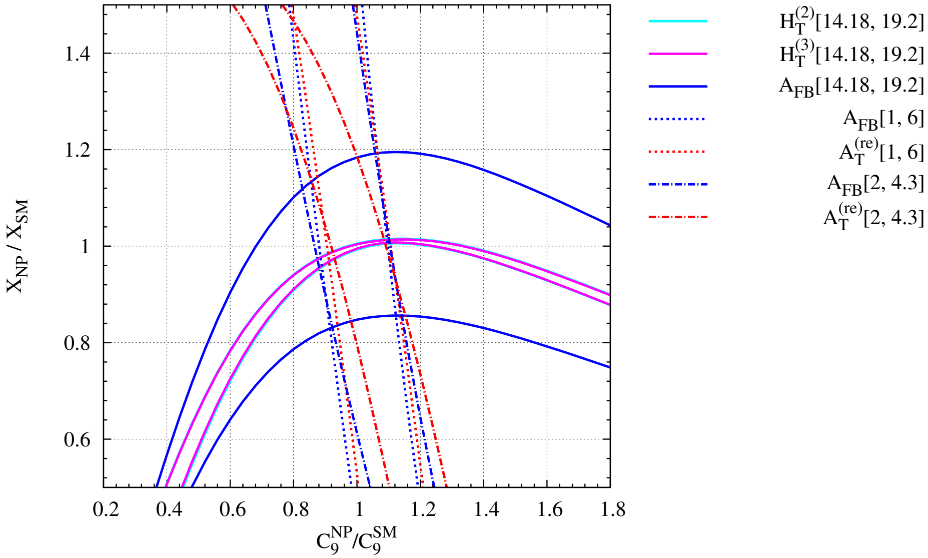
NP SENSITIVITY – REAL C_9



NP SENSITIVITY – REAL C_9



NP SENSITIVITY – REAL C_9



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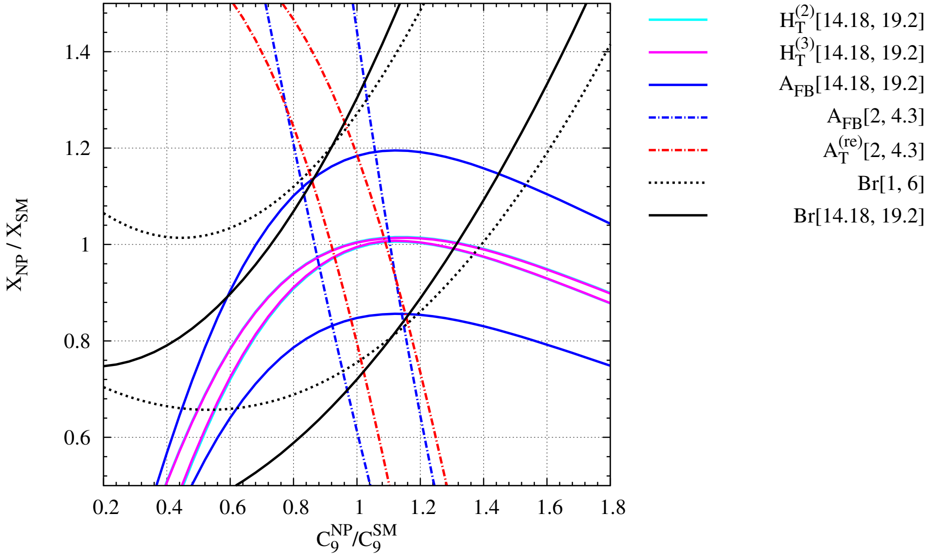
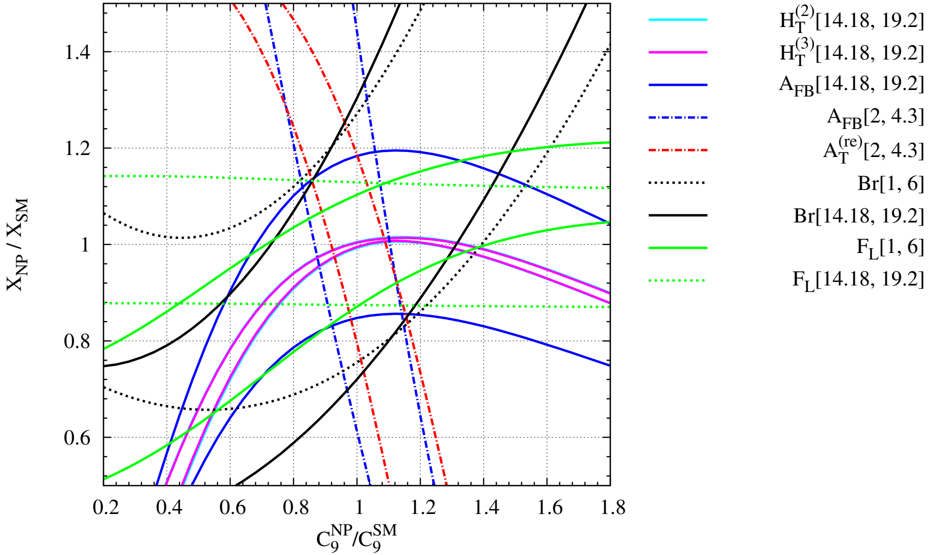
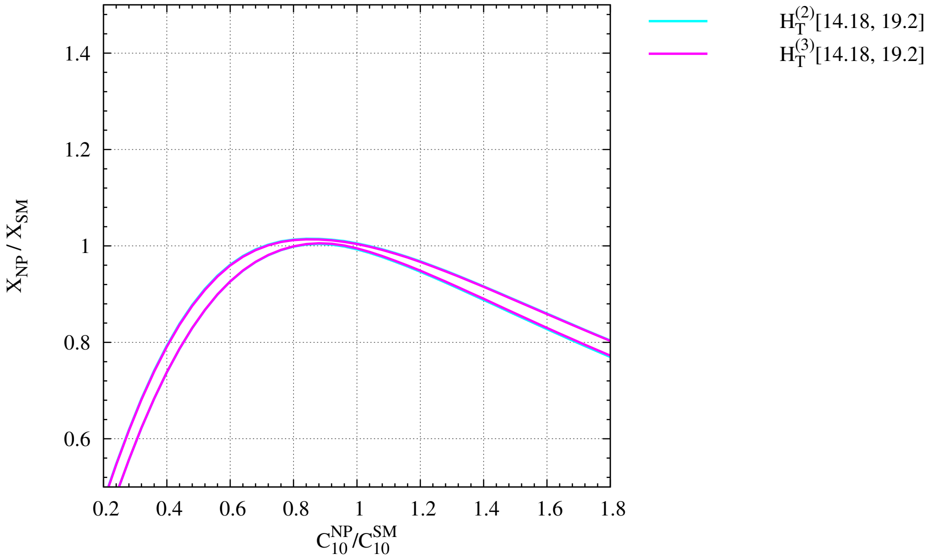


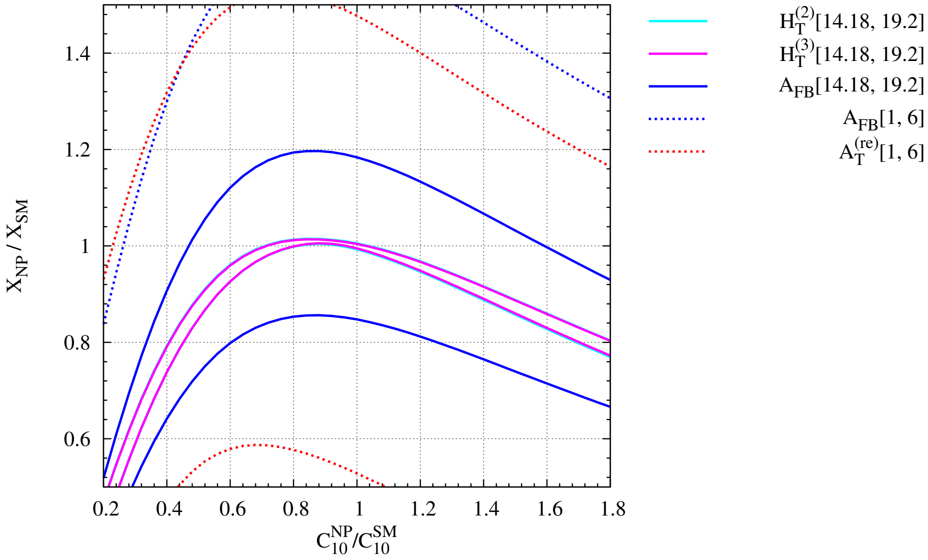
NP SENSITIVITY – REAL C_9



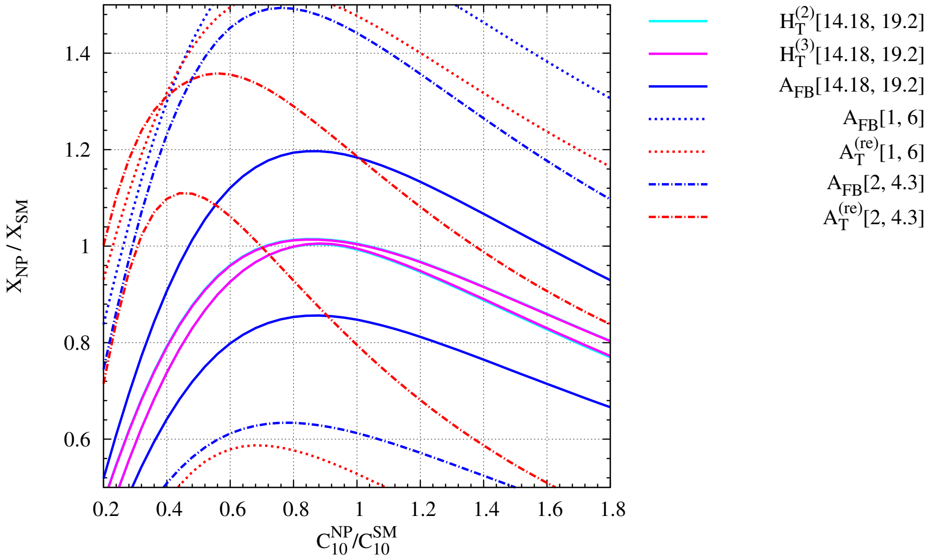
NP SENSITIVITY – REAL C_{10}



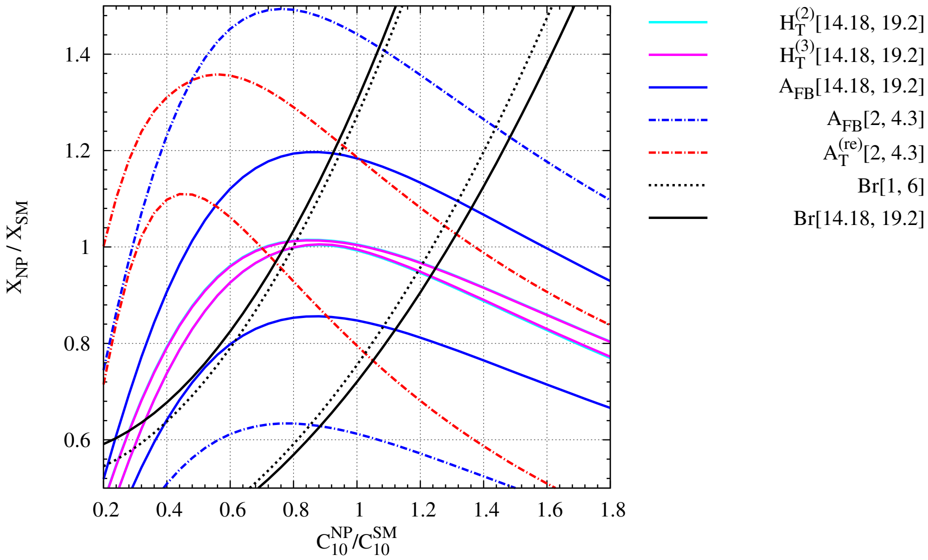
NP SENSITIVITY – REAL C_{10}



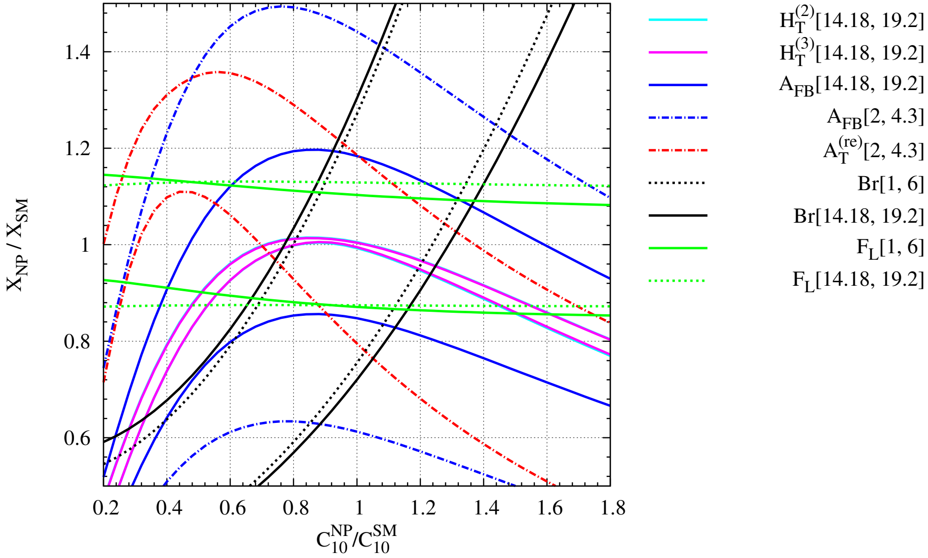
NP SENSITIVITY – REAL C_{10}



NP SENSITIVITY – REAL C_{10}



NP SENSITIVITY – REAL C_{10}



Backup Slides

q^2-INTEGRATED OBSERVABLES

Experimental measurements of observables P always imply binning in kinematical variables x, i.e.

$$\langle P \rangle_{[x_{min},\, x_{max}]} \equiv \int_{x_{min}}^{x_{max}} dx\, P(x)$$

Assume, that angular observables $I_i^{(k)}(q^2)$ are measured in experiment for certain q^2 binning (omitting q^2-interval boundaries)

$$\langle I_i^{(k)} \rangle = \int_{q^2_{min}}^{q^2_{max}} dq^2\, I_i^{(k)}(q^2)$$

and "transversity observables" are then determined as follows (for example)

$$\langle A_T^{(3)} \rangle = \sqrt{\frac{4\langle I_4 \rangle^2 + \langle I_7 \rangle^2}{-2\langle I_2^c \rangle \langle 2 I_2^s + I_3 \rangle}}$$

$\longrightarrow$ This has to accounted for in theoretical predictions !!!

form factors are cancelling → reduced hadronic uncertainties

- @ low-q^2

$$A_T^{(2)} = \frac{I_3}{2 I_2^s}, \qquad A_T^{(\mathrm{re})} = \frac{I_6^s}{4 I_2^s}, \qquad A_T^{(\mathrm{im})} = \frac{I_9}{2 I_2^s}$$

$$A_T^{(3)} = \sqrt{\frac{(2 I_4)^2 + (I_7)^2}{-2 I_2^c \left(2 I_2^s + I_3\right)}}, \qquad A_T^{(4)} = \sqrt{\frac{(I_5)^2 + (2 I_8)^2}{(2 I_4)^2 + (I_7)^2}},$$

[Krüger/Matias hep-ph/0502060, Egede/Hurth/Matias/Ramon/Reece arXiv:0807.2589 + 1005.0571, Becirevic/Schneider, arXiv:1106.3283]

- @ high-q^2

$$H_T^{(2)} = \frac{I_5}{\sqrt{-2 I_2^c \left(2 I_2^s + I_3\right)}}, \qquad H_T^{(3)} = \frac{I_6^s}{2\sqrt{(2 I_2^s)^2 - (I_3)^2}},$$

$$H_T^{(4)} = \frac{\sqrt{2} I_8}{\sqrt{-I_2^c \left(2 I_2^s + I_3\right)}}, \qquad H_T^{(5)} = \frac{I_9}{\sqrt{(2 I_2^s)^2 - (I_3)^2}}$$

[CB/Hiller/van Dyk arXiv:1006.5013 + in prep]

Measuring Angular Observables

likely that exp. results only in some q^2-integrated bins: $\langle \ldots \rangle = \int_{q^2_{min}}^{q^2_{max}} dq^2 \ldots$,
then use some (quasi-) single-diff. distributions in θ_ℓ, θ_{K*}, ϕ

- $$\frac{d\langle\Gamma\rangle}{d\phi} = \frac{1}{2\pi}\{\langle\Gamma\rangle + \langle I_3\rangle \cos 2\phi + \langle I_9\rangle \sin 2\phi\}$$

- 2 bins in $\cos\theta_{K*}$

$$\frac{d\langle A_{\theta_{K*}}\rangle}{d\phi} \equiv \int_{-1}^{1} d\cos\theta_l \left[\int_0^1 - \int_{-1}^0\right] d\cos\theta_{K*} \frac{d^3\langle\Gamma\rangle}{d\cos\theta_{K*}\, d\cos\theta_l\, d\phi}$$

$$= \frac{3}{16}\{\langle I_5\rangle \cos\phi + \langle I_7\rangle \sin\phi\}$$

- (2 bins in $\cos\theta_{K*}$) + (2 bins in $\cos\theta_l$)

$$\frac{d\langle A_{\theta_{K*},\theta_l}\rangle}{d\phi} \equiv \left[\int_0^1 - \int_{-1}^0\right] d\cos\theta_l \frac{d^2\langle A_{\theta_{K*}}\rangle}{d\cos\theta_l\, d\phi} = \frac{1}{2\pi}\{\langle I_4\rangle \cos\phi + \langle I_8\rangle \sin\phi\}$$

HIGH-q^2: OPE + HQET

Framework developed by Grinstein/Pirjol hep-ph/0404250

1) OPE in $\Lambda_{\rm QCD}/Q$ with $Q = \{m_b, \sqrt{q^2}\}$ + matching on HQET + expansion in m_c

$$\mathcal{M}[\bar{B} \to \bar{K}^* + \bar{\ell}\ell] \sim \frac{8\pi}{q^2} \sum_{i=1}^{6} C_i(\mu)\, \mathcal{T}_\alpha^{(i)}(q^2, \mu)\, [\bar{\ell}\gamma^\alpha \ell]$$

$$\mathcal{T}_\alpha^{(i)}(q^2, \mu) = i \int d^4x\, e^{iq\cdot x} \langle \bar{K}^* | T\{\mathcal{O}_i(0), j_\alpha^{\rm em}(x)\} | \bar{B} \rangle$$

$$= \sum_{k \geqslant -2} \sum_j C_{i,j}^{(k)} \langle \mathcal{Q}_{j,\alpha}^{(k)} \rangle$$

$\mathcal{Q}_{j,\alpha}^{(k)}$	power	$\mathcal{O}(\alpha_s)$
$\mathcal{Q}_{1,2}^{(-2)}$	1	$\alpha_s^0(Q)$
$\mathcal{Q}_{1-5}^{(-1)}$	$\Lambda_{\rm QCD}/Q$	$\alpha_s^1(Q)$
$\mathcal{Q}_{1,2}^{(0)}$	m_c^2/Q^2	$\alpha_s^0(Q)$
$\mathcal{Q}_{j>3}^{(0)}$	${\Lambda_{\rm QCD}}^2/Q^2$	$\alpha_s^0(Q)$
$\mathcal{Q}_i^{(2)}$	m_c^4/Q^4	$\alpha_s^0(Q)$

included,
unc. estimate by naive pwr cont.

2) HQET FF-relations at sub-leading order + α_s corrections in leading order

$$T_1(q^2) = \kappa\, V(q^2), \qquad T_2(q^2) = \kappa\, A_1(q^2), \qquad T_3(q^2) = \kappa\, A_2(q^2) \frac{M_B^2}{q^2},$$

$$\kappa = \left(1 + \frac{2 D_0^{(v)}(\mu)}{C_0^{(v)}(\mu)}\right) \frac{m_b(\mu)}{M_B}$$

can express everything in terms of QCD FF's $V, A_{1,2}$ @ $\mathcal{O}(\alpha_s \Lambda_{\rm QCD}/Q)$!!!

EXCLUSIVE OBSERVABLES (I)

$\bar{B} \to \bar{K}^* \ell^+ \ell^-$

HI-q^2 $\mathcal{B}$, A_{FB}, F_{L}, $A_{\mathrm{T}}^{(i)}$, $H_{\mathrm{T}}^{(i)}$, $a_{\mathrm{CP}}^{(i)}$

all observables: q^2-integrated and single-differential in q^2
calculation according to C. Bobeth,G. Hiller,DvD '10

$\bar{B} \to \bar{K} \ell^+ \ell^-$

HI-q^2 $\mathcal{B}$, F_{H}, $R_K^{\mu/e}$, $a_{\mathrm{CP}}^{(1)}$

all observables: q^2-integrated $\mathcal{B}$, F_{H}: also single-differential in q^2
calculation according to C. Bobeth,G. Hiller,DvD,C. Wacker '11

EXCLUSIVE OBSERVABLES (II)

$\bar{B} \to \bar{K}^* \ell^+ \ell^-$

LO-q^2 $\mathcal{B}$, A_{FB}, F_{L}, $A_{\mathrm{T}}^{(i)}$

all observables: q^2-integrated and single-differential in q^2
calculation according to M. Beneke,Th. Feldmann,D. Seidel '01 and '04

$\bar{B} \to \bar{K} \ell^+ \ell^-$

LO-q^2 $\mathcal{B}$, F_{H}, $R_K^{\mu/e}$

all observables: q^2-integrated $\mathcal{B}$, F_{H}: also single-differential in q^2
calculation according to M. Beneke,Th. Feldmann,D. Seidel '01 and '04

EXCLUSIVE OBSERVABLES (III)

$\bar{B} \to \bar{K}^* \gamma$

$\mathcal{B}$, $S_{K*\gamma}$, $C_{K*\gamma}$

calculation according to M. Beneke,Th. Feldmann,D. Seidel '01 and '04 for $q^2 \to 0$

$\bar{B}_{s,d} \to \ell^+ \ell^-$

$\mathcal{B}$

calculation according to C. Bobeth,T. Ewerth,F. Krüger,J. Urban '02

[Also: $\bar{B} \to X_s \ell^+ \ell^-$ is implemented for the SM Basis only.]