

$$b \rightarrow s + \bar{\ell}\ell$$

PHENOMENOLOGY

@ HIGH- q^2

Christoph Bobeth

TU München (IAS + Excellence Cluster Universe)

Rare B decays @ low recoil (bsll2011)

DESY - Hamburg

OUTLINE

1) Effective theory (EFT) of $\Delta B = 1$ FCNC decays

A) In the Standard Model (SM)

B) Beyond the SM (BSM)

2) Exclusive $B \rightarrow V_{on-shell} (\rightarrow P_1 P_2) + \bar{\ell}\ell$

A) Kinematics and observables in angular distribution

B) High- q^2 : SM Op's-basis + Fit

C) High- q^2 : BSM

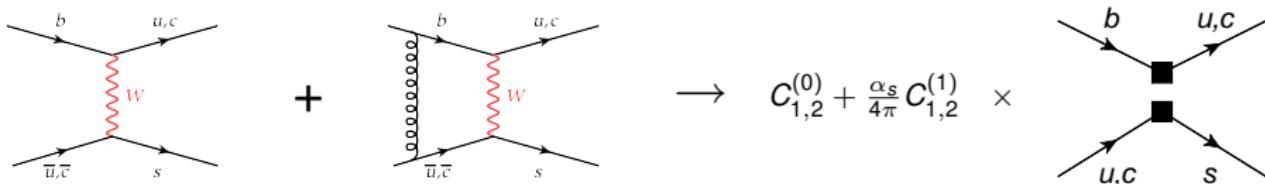
3) Exclusive $B \rightarrow P + \bar{\ell}\ell$

EFT of $\Delta B = 1$ decays in SM and beyond

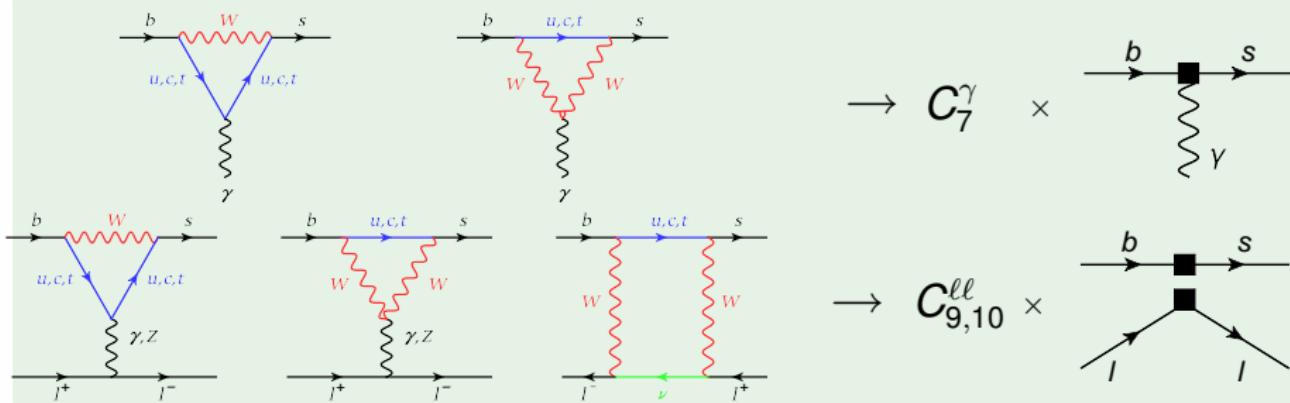
$\Delta B = 1$ EFT IN THE SM (FOR $b \rightarrow s$)

- I) decoupling (OPE) of heavy particles (W, Z, t, \dots) @ EW scale: $\mu_{EW} \gtrsim M_W$
 → factorisation into short-distance: C_i and long-distance: \mathcal{O}_i

- II) RG-running to lower scale: $\mu_b \sim m_b$ → resums large log's: $[\alpha_s \ln(\mu_b/\mu_{EW})]^n$



MOST RELEVANT FOR $b \rightarrow s + \bar{\ell}\ell$



SM OPERATOR LIST

... USING CKM UNITARITY

$$\mathcal{L}_{\text{SM}} \sim \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\mathcal{L}_{\text{SM}}^{(t)} + \hat{\lambda}_u \mathcal{L}_{\text{SM}}^{(u)} \right), \quad \hat{\lambda}_u = V_{ub} V_{us}^* / V_{tb} V_{ts}^*$$

$$\mathcal{L}_{\text{SM}}^{(u)} = C_1(\mathcal{O}_1^c - \mathcal{O}_1^u) + C_2(\mathcal{O}_2^c - \mathcal{O}_2^u)$$

$$\mathcal{O}_{1,2}^{u,c} = \text{curr.-curr.: } b \rightarrow s \{ \bar{u}u, \bar{c}c \}$$

\Rightarrow CP-violation in the SM is tiny

$$\text{Im}[\hat{\lambda}_u] \approx \lambda^2 \bar{\eta} \sim 10^{-2}$$

$$\mathcal{L}_{\text{SM}}^{(t)} = C_1 \mathcal{O}_1^c + C_2 \mathcal{O}_2^c + \sum_{i>2} C_i \mathcal{O}_i$$

$$\mathcal{O}_7^\gamma = \text{electr.magn.} \quad b \rightarrow s \gamma$$

$$\mathcal{O}_{9,10}^{\ell\ell} = \text{semi-lept.} \quad b \rightarrow s \ell\bar{\ell}$$

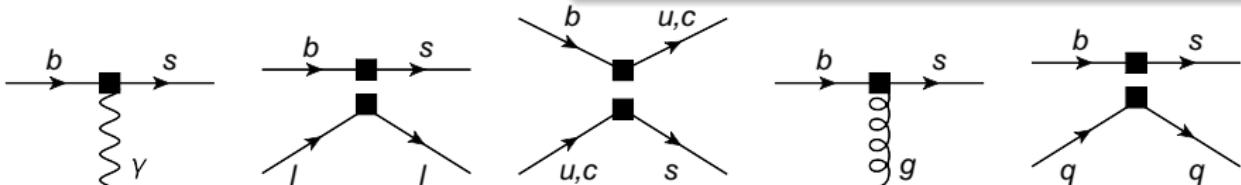
$$\mathcal{O}_{1,2}^c = \text{curr.-curr.} \quad b \rightarrow s \bar{c}c$$

$$\mathcal{O}_8^g = \text{chromo.magn.} \quad b \rightarrow s g$$

$$\mathcal{O}_{3,4,5,6} = \text{QCD-peng.} \quad b \rightarrow s \bar{q}q, q = \{u, d, s, c, b\}$$

$$\mathcal{O}_{3,4,5,6}^Q = \text{QED-peng.} \quad b \rightarrow s \bar{q}q, q = \{u, d, s, c, b\}$$

$$\mathcal{O}_b = \text{QED-box} \quad b \rightarrow s \bar{b}b$$



GENERAL APPROACH BEYOND SM ...

- MODEL-DEP.
- 1) decoupling of new heavy particles @ NP scale: $\mu_{NP} \gtrsim M_W$
 - 2) RG-running to lower scale $\mu_b \sim m_b$ (potentially tower of EFT's)

MODEL-INDEP. extending SM EFT-Lagrangian → ...

... beyond the SM:

- ⇒ ??? ... additional light degrees of freedom (\Leftarrow not pursued in the following)
- ⇒ ΔC_i ... NP contributions to SM C_i
- ⇒ $\sum_{NP} C_j \mathcal{O}_j(???)$... NP operators (e.g. $C'_{7,9,10}$, $C'_{S,P}$, ...)

$$\mathcal{L}_{EFT}(\mu_b) = \mathcal{L}_{QED \times QCD}(u, d, s, c, b, e, \mu, \tau, ???)$$

$$+ \frac{4G_F}{\sqrt{2}} V_{CKM} \sum_{SM} (C_i + \Delta C_i) \mathcal{O}_i + \sum_{NP} C_j \mathcal{O}_j(???)$$

BEYOND THE SM OPERATOR LIST

FREQUENTLY CONSIDERED IN MODEL-(IN)DEPENDENT SEARCHES: $b \rightarrow s + \bar{\ell}\ell$

SM' = χ -flipped SM analogues

$$\mathcal{O}_{7',8'}^{\gamma,g} = \frac{(e, g_s)}{16\pi^2} m_b [\bar{s} \sigma_{\mu\nu} P_L(T^a) b] (F, G^a)^{\mu\nu}, \quad \mathcal{O}_{9',10'}^{\ell\ell} = \frac{\alpha_e}{4\pi} [\bar{s} \gamma^\mu P_R b] [\bar{\ell} (\gamma^\mu, \gamma^\mu \gamma_5) \ell],$$

S + P = scalar + pseudoscalar

$$\mathcal{O}_{S,S'}^{\ell\ell} = \frac{\alpha_e}{4\pi} [\bar{s} P_{R,L} b] [\bar{\ell} \ell], \quad \mathcal{O}_{P,P'}^{\ell\ell} = \frac{\alpha_e}{4\pi} [\bar{s} P_{R,L} b] [\bar{\ell} \gamma_5 \ell],$$

T = tensor

$$\mathcal{O}_T^{\ell\ell} = \frac{\alpha_e}{4\pi} [\bar{s} \sigma_{\mu\nu} b] [\bar{\ell} \sigma^{\mu\nu} \ell], \quad \mathcal{O}_{TE}^{\ell\ell} = \frac{\alpha_e}{4\pi} i \varepsilon^{\mu\nu\alpha\beta} [\bar{s} \sigma_{\mu\nu} b] [\bar{\ell} \sigma_{\alpha\beta} \ell],$$

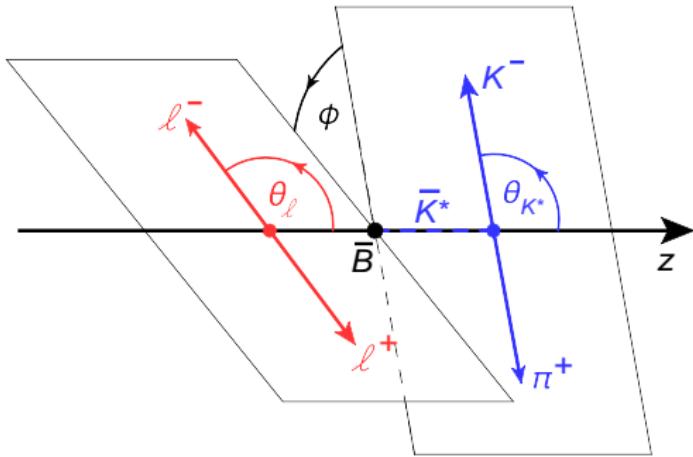
- Dirac-structures BSM: right-handed currents, (pseudo-) scalar and/or tensor interactions
- usually added to $\mathcal{L}_{\text{SM}}^{(t)}$

⇒ EFT starting point for calculation of observables
!!! Non-PT input required when evaluating matrix elements

$$B \rightarrow V_{on-shell}(\rightarrow P_1 P_2) + \bar{\ell} \ell$$

KINEMATICS

- for on-resonance V decays
→ narrow width approximation
→ 4 kinematic variables
(off-reson. 5 kin. variables)
- $Br(K^* \rightarrow K\pi) \approx 99\%$
- $\bar{B}^0 \rightarrow \bar{K}^{*0} (\rightarrow K^-\pi^+, \bar{K}^0\pi^0) + \bar{\ell}\ell$
and CP-conjugated decay:
 $B^0 \rightarrow K^{*0} (\rightarrow K^+\pi^-, K^0\pi^0) + \bar{\ell}\ell$
- similarly $B_s \rightarrow \phi (\rightarrow K^+K^-) + \bar{\ell}\ell$



$$\bar{B}^0(p_B) \rightarrow \bar{K}_{on-shell}^{*0}(p_{K^*}) [\rightarrow K^-(p_K) + \pi^+(p_\pi)] + \bar{\ell}(p_{\bar{\ell}}) + \ell(p_\ell)$$

- $q^2 = m_{\bar{\ell}\ell}^2 = (p_{\bar{\ell}} + p_\ell)^2 = (p_B - p_{K^*})^2$ $4m_\ell^2 \leq q^2 \leq (M_B - M_{K^*})^2$
- $\cos \theta_\ell$ with $\theta_\ell \angle (\vec{p}_B, \vec{p}_{\bar{\ell}})$ in $(\bar{\ell}\ell)$ -c.m. system $-1 \leq \cos \theta_\ell \leq 1$
- $\cos \theta_{K^*}$ with $\theta_{K^*} \angle (\vec{p}_B, \vec{p}_K)$ in $(K\pi)$ -c.m. system $-1 \leq \cos \theta_{K^*} \leq 1$
- $\phi \angle (\vec{p}_K \times \vec{p}_\pi, \vec{p}_{\bar{\ell}} \times \vec{p}_\ell)$ in B -RF $-\pi \leq \phi \leq \pi$

ANGULAR DISTRIBUTION

DIFF. ANGULAR DISTRIBUTION

$$\frac{32\pi}{9} \frac{d^4\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_{K^*} d\phi} = I_1^S \sin^2\theta_{K^*} + I_1^C \cos^2\theta_{K^*} + (I_2^S \sin^2\theta_{K^*} + I_2^C \cos^2\theta_{K^*}) \cos 2\theta_\ell \\ + I_3 \sin^2\theta_{K^*} \sin^2\theta_\ell \cos 2\phi + I_4 \sin 2\theta_{K^*} \sin 2\theta_\ell \cos\phi + I_5 \sin 2\theta_{K^*} \sin\theta_\ell \cos\phi \\ + (I_6^S \sin^2\theta_{K^*} + I_6^C \cos^2\theta_{K^*}) \cos\theta_\ell + I_7 \sin 2\theta_{K^*} \sin\theta_\ell \sin\phi \\ + I_8 \sin 2\theta_{K^*} \sin 2\theta_\ell \sin\phi + I_9 \sin^2\theta_{K^*} \sin^2\theta_\ell \sin 2\phi$$

$I_i^{(k)}(q^2)$ = q^2 -dependent “ANGULAR OBSERVABLES”

$\Rightarrow 2 \times (12 + 12) = 48$ when measuring separately

A) decay + CP-conjugate decay

B) for each $\ell = e, \mu$ (τ 's are interesting too!!!)

CP-conjugated decay: $d^4\bar{\Gamma}$ from $d^4\Gamma$ by replacing

$$I_{1,2,3,4,7}^{(k)} \rightarrow + \bar{I}_{1,2,3,4,7}^{(k)} [\delta_W \rightarrow -\delta_W], \quad \text{CP-even}$$

$$I_{5,6,8,9}^{(k)} \rightarrow - \bar{I}_{5,6,8,9}^{(k)} [\delta_W \rightarrow -\delta_W], \quad \text{CP-odd}$$

with $\ell \leftrightarrow \bar{\ell} \Rightarrow \theta_\ell \rightarrow \theta_\ell - \pi$ and $\phi \rightarrow -\phi$ and weak phases δ_W conjugated

OBSERVABLES - I

- for (SM + χ -flipped) operators and $m_\ell = 0$: $I_1^S = 3I_2^S$, $I_1^C = -I_2^C$, $I_6^C = 0$
- in presence of scalar and/or tensor operators: $I_6^C \neq 0$

COMBINING DECAY + CP-CONJUGATED DECAY

CP-averaged $S_i^{(k)} = [I_i^{(k)} + \bar{I}_i^{(k)}] / \frac{d(\Gamma + \bar{\Gamma})}{dq^2}$

CP asymmetries $A_i^{(k)} = [I_i^{(k)} - \bar{I}_i^{(k)}] / \frac{d(\Gamma + \bar{\Gamma})}{dq^2}$

- normalisation to CP-ave rate → reduce form factor dependence
BUT better suited normalisations possible (examples later)
- if full angular fit from experimental data possible then
 - 1) $S_{1,2,3,4,7}^{(k)}$ and $A_{5,6,8,9}^{(k)}$ from $d^4(\Gamma + \bar{\Gamma})$ = flavour-untagged B samples
 - 2) $A_{1,2,3,4,7}^{(k)}$ and $S_{5,6,8,9}^{(k)}$ from $d^4(\Gamma - \bar{\Gamma})$

CP-odd ($i = 5,6,8,9$) \Rightarrow CP-asymmetries $\sim d^4(\Gamma + \bar{\Gamma})$
can be measured from untagged (equally mixed ???) B samples
??? requires knowledge of \bar{B}/B -fraction of untagged sample: LHCb vs SuperB

OBSERVABLES - II

- decay rate $\frac{d\Gamma}{dq^2} = \frac{3}{4}(2I_1^s + I_1^c) - \frac{1}{4}(2I_2^s + I_2^c)$, $\frac{d\bar{\Gamma}}{dq^2} = \frac{d\Gamma}{dq^2}[I_i^{(k)} \rightarrow \bar{I}_i^{(k)}]$
- rate CP-asymmetry

$$A_{\text{CP}} = \frac{d(\Gamma - \bar{\Gamma})}{dq^2} / \frac{d(\Gamma + \bar{\Gamma})}{dq^2} = \frac{3}{4}(2A_1^s + A_1^c) - \frac{1}{4}(2A_2^s + A_2^c)$$

- lepton forward-backward asymmetry

$$A_{\text{FB}} = \left[\int_0^1 - \int_{-1}^0 \right] d \cos \theta_\ell \frac{d^2(\Gamma - \bar{\Gamma})}{dq^2 d \cos \theta_\ell} / \frac{d(\Gamma + \bar{\Gamma})}{dq^2} = \frac{3}{8}(2S_6^s + S_6^c)$$

- lepton forward-backward CP-asymmetry

$$A_{\text{FB}}^{\text{CP}} = \left[\int_0^1 - \int_{-1}^0 \right] d \cos \theta_\ell \frac{d^2(\Gamma + \bar{\Gamma})}{dq^2 d \cos \theta_\ell} / \frac{d(\Gamma + \bar{\Gamma})}{dq^2} = \frac{3}{8}(2A_6^s + A_6^c)$$

- CP-ave. longitudinal and transverse K^* polarisation fractions

$$F_L = -S_2^c,$$

$$F_T = 4S_2^s$$

OBSERVABLES - III

- “transversity observables” (designed for low- q^2)

$$A_T^{(2)} = \frac{S_3}{2S_2^s}, \quad A_T^{(3)} = \sqrt{\frac{4S_4^2 + S_7^2}{-2S_2^c(2S_2^s + S_3)}}, \quad A_T^{(4)} = \sqrt{\frac{S_5^2 + 4S_8^2}{4S_4^2 + S_7^2}}$$

- lepton-flavour e, μ -non-universal (extend to $I_i^{(k)}$)

$$R_{K^*(X_s, K)} = \frac{d\Gamma[B \rightarrow K^*(X_s, K) + \bar{e}e]}{dq^2} / \frac{d\Gamma[B \rightarrow K^*(X_s, K) + \bar{\mu}\mu]}{dq^2}$$

- isospin asymmetry (extend to $I_i^{(k)}$) - only @ low- q^2 , @ high- $q^2 \sim 1/m_b^3$)

$$A_I = \frac{(\tau_{B^+}/\tau_{B^0}) \times dBr[B^0 \rightarrow K^{*0} \bar{\ell}\ell] - dBr[B^+ \rightarrow K^{*+} \bar{\ell}\ell]}{(\tau_{B^+}/\tau_{B^0}) \times dBr[B^0 \rightarrow K^{*0} \bar{\ell}\ell] + dBr[B^+ \rightarrow K^{*+} \bar{\ell}\ell]}$$

- and others... $A_T^{(5)}, A_{6s}^{V2s}, A_8^V, H_T^{(1,2,3)} \dots$

MEASURING ANGULAR OBSERVABLES

likely that exp. results only in some q^2 -integrated bins: $\langle \dots \rangle = \int_{q^2_{min}}^{q^2_{max}} dq^2 \dots$,
then use some (quasi-) single-diff. distributions in θ_ℓ , θ_{K^*} , ϕ

-

$$\frac{d \langle \Gamma \rangle}{d\phi} = \frac{1}{2\pi} \{ \langle \Gamma \rangle + \langle I_3 \rangle \cos 2\phi + \langle I_9 \rangle \sin 2\phi \}$$

- 2 bins in $\cos \theta_{K^*}$

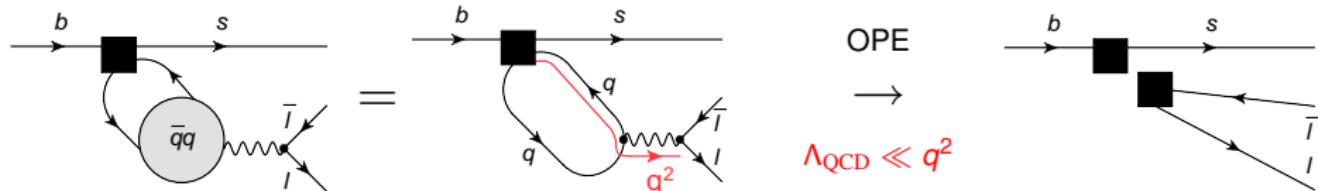
$$\begin{aligned} \frac{d \langle A_{\theta_{K^*}} \rangle}{d\phi} &\equiv \int_{-1}^1 d \cos \theta_I \left[\int_0^1 - \int_{-1}^0 \right] d \cos \theta_{K^*} \frac{d^3 \langle \Gamma \rangle}{d \cos \theta_{K^*} d \cos \theta_I d\phi} \\ &= \frac{3}{16} \{ \langle I_5 \rangle \cos \phi + \langle I_7 \rangle \sin \phi \} \end{aligned}$$

- (2 bins in $\cos \theta_{K^*}$) + (2 bins in $\cos \theta_I$)

$$\frac{d \langle A_{\theta_{K^*}, \theta_I} \rangle}{d\phi} \equiv \left[\int_0^1 - \int_{-1}^0 \right] d \cos \theta_I \frac{d^2 \langle A_{\theta_{K^*}} \rangle}{d \cos \theta_I d\phi} = \frac{1}{2\pi} \{ \langle I_4 \rangle \cos \phi + \langle I_8 \rangle \sin \phi \}$$

HIGH- q^2 : OPE – I

Hard momentum transfer ($q^2 \sim M_B^2$) through $(\bar{q}q) \rightarrow \bar{\ell}\ell$ allows local OPE



$$\begin{aligned} \mathcal{M}[\bar{B} \rightarrow \bar{K}^* + \bar{\ell}\ell] &\sim \frac{8\pi^2}{q^2} i \int d^4x e^{iq \cdot x} \langle \bar{K}^* | T\{\mathcal{L}^{\text{eff}}(0), j_\mu^{\text{em}}(x)\} | \bar{B} \rangle [\bar{\ell} \gamma^\mu \ell] \\ &= \left(\sum_a C_{3a} Q_{3a}^\mu + \sum_b C_{5b} Q_{5b}^\mu + \sum_c C_{6c} Q_{6c}^\mu + \mathcal{O}(\text{dim} > 6) \right) [\bar{\ell} \gamma_\mu \ell] \end{aligned}$$

Buchalla/Isidori hep-ph/9801456, Grinstein/Pirjol hep-ph/0404250, Beylich/Buchalla/Feldmann arXiv:1101.5118

Leading $\text{dim} = 3$ operators: $\langle \bar{K}^* | Q_{3,a} | \bar{B} \rangle \sim \text{usual } B \rightarrow K^* \text{ form factors } V, A_{0,1,2}, T_{1,2,3}$

$$Q_{3,1}^\mu = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) [\bar{s} \gamma_\nu (1 - \gamma_5) b] \quad \rightarrow \quad C_9 \rightarrow C_9^{\text{eff}}, \quad (V, A_{0,1,2})$$

$$Q_{3,2}^\mu = \frac{im_b}{q^2} q_\nu [\bar{s} \sigma_{\nu\mu} (1 + \gamma_5) b] \quad \rightarrow \quad C_7 \rightarrow C_7^{\text{eff}}, \quad (T_{1,2,3})$$

HIGH- q^2 : OPE – II

$\text{dim} = 3$ α_s matching corrections are also known

$m_s \neq 0$ 2 additional $\text{dim} = 3$ operators, suppressed with $\alpha_s m_s / m_b \sim 0.5\%$,
NO new form factors

$\text{dim} = 4$ absent

$\text{dim} = 5$ suppressed by $(\Lambda_{\text{QCD}} / m_b)^2 \sim 2\%$,
explicite estimate @ $q^2 = 15 \text{ GeV}^2$: < 1% [Beylich/Buchalla/Feldmann arXiv:1101.5118]

$\text{dim} = 6$ suppressed by $(\Lambda_{\text{QCD}} / m_b)^3 \sim 0.2\%$ and small QCD-penguin's: $C_{3,4,5,6}$
spectator quark effects: from weak annihilation

BEYOND OPE duality violating effects [Beylich/Buchalla/Feldmann arXiv:1101.5118]

- based on Shifman model for c -quark correlator + fit to recent BES data
- $\pm 2\%$ for integrated rate $q^2 > 15 \text{ GeV}^2$

⇒ OPE of exclusive $\bar{B} \rightarrow \bar{K}^*(\bar{K}) + \bar{\ell}\ell$ predicts small sub-leading contributions !!!

BUT, still missing $B \rightarrow K^*$ form factors @ high- q^2
for predictions of angular observables $I_i^{(K)}$

ANY other effects to consider ???

HIGH- q^2 : OPE + HQET – I

Framework developed by Grinstein/Pirjol hep-ph/0404250

- 1) OPE in Λ_{QCD}/Q with $Q = \{m_b, \sqrt{q^2}\}$ + matching on HQET + expansion in m_c

$$\mathcal{M}[\bar{B} \rightarrow \bar{K}^* + \bar{\ell}\ell] \sim \frac{8\pi}{q^2} \sum_{i=1}^6 C_i(\mu) \mathcal{T}_\alpha^{(i)}(q^2, \mu) [\bar{\ell}\gamma^\alpha \ell]$$

$$\begin{aligned} \mathcal{T}_\alpha^{(i)}(q^2, \mu) &= i \int d^4x e^{iq \cdot x} \langle \bar{K}^* | T\{\mathcal{O}_i(0), j_\alpha^{\text{em}}(x)\} | \bar{B} \rangle \\ &= \sum_{k \geq -2} \sum_j C_{i,j}^{(k)} \langle Q_{j,\alpha}^{(k)} \rangle \end{aligned}$$

$Q_{j,\alpha}^{(k)}$	power	$\mathcal{O}(\alpha_s)$
$Q_{1,2}^{(-2)}$	1	$\alpha_s^0(Q)$
$Q_{1-5}^{(-1)}$	Λ_{QCD}/Q	$\alpha_s^1(Q)$
$Q_{1,2}^{(0)}$	m_c^2/Q^2	$\alpha_s^0(Q)$
$Q_{j>3}^{(0)}$	$\Lambda_{\text{QCD}}^2/Q^2$	$\alpha_s^0(Q)$
$Q_i^{(2)}$	m_c^4/Q^4	$\alpha_s^0(Q)$

included,
unc. estimate by naive pwr cont.

- 2) HQET FF-relations at sub-leading order + α_s corrections in leading order

$$T_1(q^2) = \kappa V(q^2), \quad T_2(q^2) = \kappa A_1(q^2), \quad T_3(q^2) = \kappa A_2(q^2) \frac{M_B^2}{q^2},$$

$$\kappa = \left(1 + \frac{2D_0^{(V)}(\mu)}{C_0^{(V)}(\mu)} \right) \frac{m_b(\mu)}{M_B}$$

can express everything in terms of QCD FF's $V, A_{1,2}$ @ $\mathcal{O}(\alpha_s \Lambda_{\text{QCD}}/Q)$!!!

HIGH- q^2 – AMPLITUDE STRUCTURE

TRANSVERSITY AMPLITUDES $A_i^{L,R}(\bar{B} \rightarrow \bar{K}^* \bar{\ell} \ell)$

$$A_{\perp}^{L,R} = + \left[C^{L,R} + \tilde{r}_a \right] f_{\perp}, \quad A_{\parallel}^{L,R} = - \left[C^{L,R} + \tilde{r}_b \right] f_{\parallel},$$

$$A_0^{L,R} = - C^{L,R} f_0 - NM_B \frac{(1 - \hat{s} - \hat{M}_{K^*}^2)(1 + \hat{M}_{K^*})^2 \tilde{r}_b A_1 - \hat{\lambda} \tilde{r}_c A_2}{2 \hat{M}_{K^*} (1 + \hat{M}_{K^*}) \sqrt{\hat{s}}}$$

\Rightarrow Universal short-distance coefficients: $C^{L,R} = C_9^{\text{eff}} + \kappa \frac{2m_b M_B}{q^2} C_7^{\text{eff}} \mp C_{10}$
 (SM: $C_9 \sim +4$, $C_{10} \sim -4$, $C_7 \sim -0.3$)

known structure of sub-leading corrections (Grinstein/Pirjol hep-ph/0404250)

$$\tilde{r}_i \sim \pm \frac{\Lambda_{\text{QCD}}}{m_b} \left(C_7^{\text{eff}} + \alpha_s(\mu) e^{i\delta_i} \right), \quad i = a, b, c$$

Non-PT FF's ("helicity FF's" Bharucha/Feldmann/Wick arXiv:1004.3249)

$$f_{\perp} = \frac{\sqrt{2\hat{\lambda}}}{1 + \hat{M}_{K^*}} V, \quad f_{\parallel} = \sqrt{2} (1 + \hat{M}_{K^*}) A_1, \quad f_0 = \frac{(1 - \hat{s} - \hat{M}_{K^*}^2)(1 + \hat{M}_{K^*})^2 A_1 - \hat{\lambda} A_2}{2 \hat{M}_{K^*} (1 + \hat{M}_{K^*}) \sqrt{\hat{s}}}$$

HIGH- q^2 – SM OPERATOR BASIS

ANGULAR OBSERVABLES ($m_\ell = 0$)

$$(2 I_2^s + I_3) = 2 \rho_1 f_\perp^2, \quad -I_2^c = 2 \rho_1 f_0^2, \quad I_5/\sqrt{2} = 4 \rho_2 f_0 f_\perp,$$

$$(2 I_2^s - I_3) = 2 \rho_1 f_\parallel^2, \quad \sqrt{2} I_4 = 2 \rho_1 f_0 f_\parallel, \quad I_6^s/2 = 4 \rho_2 f_\parallel f_\perp,$$

$$I_7 = I_8 = I_9 = 0, \quad (I_6^c = 0)$$

$$\rho_1 = \frac{1}{2} \left(|C^R|^2 + |C^L|^2 \right) = \left| C_9^{\text{eff}} + \kappa \frac{2m_b^2}{q^2} C_7^{\text{eff}} \right|^2 + |C_{10}|^2 = N_{\text{eff}} \text{ (Grinstein/Pirjol)},$$

$$\rho_2 = \frac{1}{4} \left(|C^R|^2 - |C^L|^2 \right) = \text{Re} \left[\left(C_9^{\text{eff}} + \kappa \frac{2m_b^2}{q^2} C_7^{\text{eff}} \right) C_{10}^* \right]$$

ρ_1 and ρ_2 are largely μ -scale independent (NNLL)

$$\frac{d\Gamma}{dq^2} = 2 \rho_1 \times (f_0^2 + f_\perp^2 + f_\parallel^2), \quad A_{\text{FB}} = 3 \frac{\rho_2}{\rho_1} \times \frac{f_\perp f_\parallel}{(f_0^2 + f_\perp^2 + f_\parallel^2)},$$

$$F_L = \frac{f_0^2}{f_0^2 + f_\perp^2 + f_\parallel^2}, \quad A_T^{(2)} = \frac{f_\perp^2 - f_\parallel^2}{f_\perp^2 + f_\parallel^2}, \quad A_T^{(3)} = \frac{f_\parallel}{f_\perp}, \quad A_T^{(4)} = 2 \frac{\rho_2}{\rho_1} \times \frac{f_\perp}{f_\parallel}$$

HIGH- q^2 – “LONG-DISTANCE FREE”

FF-FREE RATIOS

!!! TEST SD FLAVOUR COUPLINGS VERSUS EXP. DATA + OPE

$$H_T^{(1)} = \frac{\sqrt{2} I_4}{\sqrt{-I_2^c (2I_2^s - I_3)}} = 1$$

$$H_T^{(2)} = \frac{I_5}{\sqrt{-2I_2^c (2I_2^s + I_3)}} = 2 \frac{\rho_2}{\rho_1}, \quad H_T^{(3)} = \frac{I_6}{2\sqrt{(2I_2^s)^2 - I_3^2}} = 2 \frac{\rho_2}{\rho_1}$$

SM predictions integrated $q^2 \in [14, 19.2] \text{ GeV}^2$ (CB/Hiller/van Dyk arXiv:1006.5013)

$$\langle H_T^{(1)} \rangle = +0.997 \pm 0.002 \Big|_{\text{FF}} \begin{array}{c} +0.000 \\ -0.001 \end{array} \Big|_{\text{IWR}},$$

$$\langle H_T^{(2)} \rangle = -0.972 \pm 0.004 \Big|_{\text{FF}} \begin{array}{c} +0.004 \\ -0.003 \end{array} \Big|_{\text{SL}} \begin{array}{c} +0.008 \\ -0.005 \end{array} \Big|_{\text{IWR}} \begin{array}{c} +0.003 \\ -0.004 \end{array} \Big|_{\text{SD}},$$

$$\langle H_T^{(3)} \rangle = -0.958 \pm 0.001 \Big|_{\text{SL}} \begin{array}{c} +0.008 \\ -0.006 \end{array} \Big|_{\text{IWR}} \begin{array}{c} +0.003 \\ -0.004 \end{array} \Big|_{\text{SD}}$$

⇒ Assuming validity of **LCSR extrapolation** Ball/Zwicky [hep-ph/0412079] of $V, A_{1,2}(q^2)$ to $q^2 > 14 \text{ GeV}^2$ based form factor parametrisation using dipole formula

⇒ $\langle \dots \rangle = q^2$ -integration performed in analogy to experimental measurement for each $I_i^{(k)}$ before taking ratio and $\sqrt{\dots}$

HIGH- q^2 – “SHORT-DISTANCE FREE”

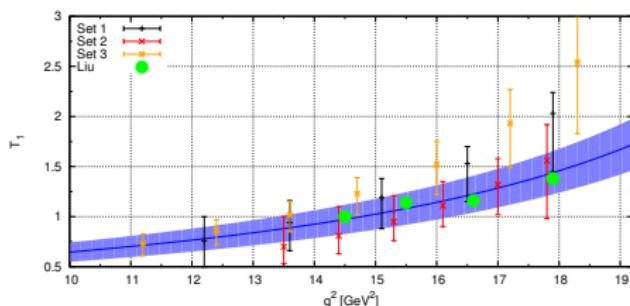
SHORT-DISTANCE-FREE RATIOS

!!! TEST LATTICE VERSUS EXP. DATA + OPE

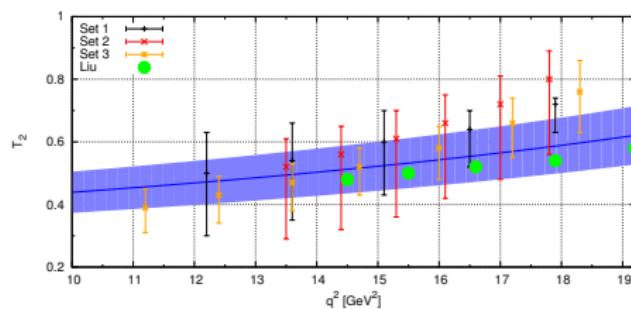
$$\frac{f_0}{f_{\parallel}} = \frac{\sqrt{2}I_5}{I_6} = \frac{-I_2^c}{\sqrt{2}I_4} = \frac{\sqrt{2}I_4}{2I_2^s - I_3} = \sqrt{\frac{-I_2^c}{2I_2^s - I_3}},$$

$$\frac{f_{\perp}}{f_{\parallel}} = \sqrt{\frac{2I_2^s + I_3}{2I_2^s - I_3}} = \frac{\sqrt{-I_2^c(2I_2^s + I_3)}}{\sqrt{2}I_4},$$

$$\frac{f_0}{f_{\perp}} = \sqrt{\frac{-I_2^c}{2I_2^s + I_3}}$$



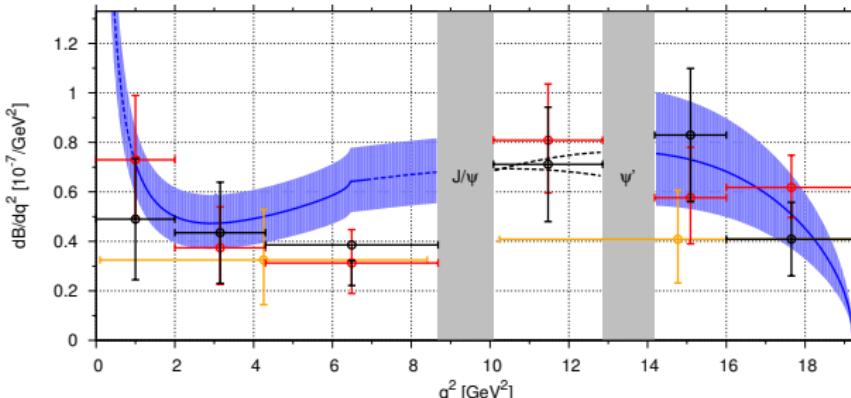
LCSR extrapolation (Ball/Zwicky hep-ph/0412079) of $T_1(q^2)$ and $T_2(q^2)$ to high- q^2 versus quenched Lattice (3 data sets from Becirevic/Lubicz/Mescia hep-ph/0611295)



new unquenched Lattice results to come →
Liu/Meinel/Hart/Horgan/Müller/Wingate
arXiv:0911.2370, arXiv:1101.2726
no final uncertainty estimate yet

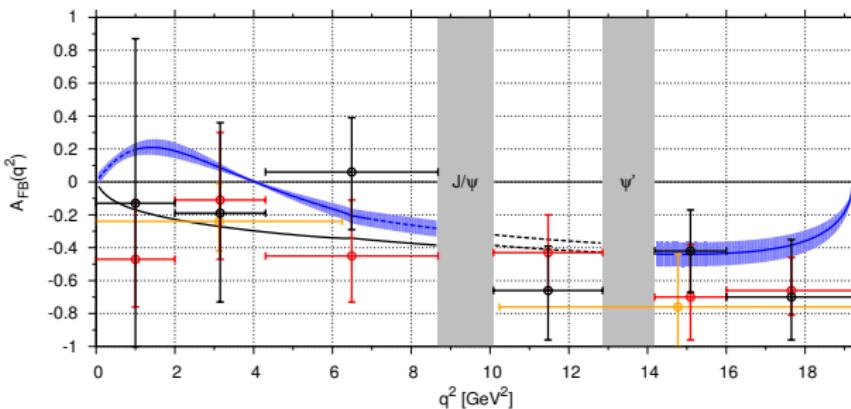
NO lattice results yet for $B \rightarrow K^*$ FF's @ high- q^2 : $V, A_{0,1,2}, T_3$!!!

HIGH- q^2 – Br , A_{fb}



Br and A_{FB}

SM prediction + unc.
@ low- and high- q^2



Data points from

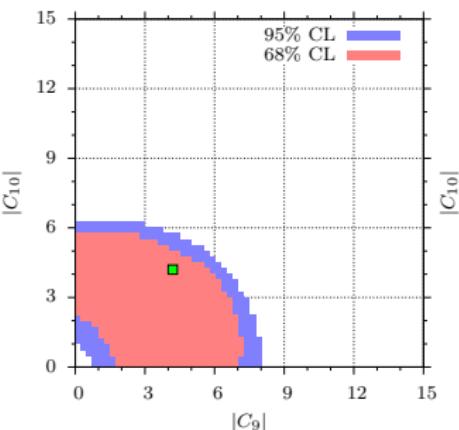
[Babar '08]

[Belle '09]

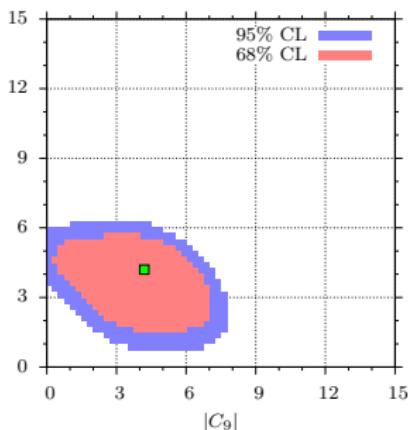
[CDF '10]

“GLOBAL” FIT OF C_9 AND C_{10} – COMPLEX

Large Recoil + Inclusive



All Data



CB/Hiller/van Dyk arXiv:1105.0376

Scan resolution

$$|C_7| \in [.30, .35], \quad \Delta|C_7| = .01$$

$$|C_{9,10}| \in [0, 15], \quad \Delta|C_{9,10}| = 0.25$$

$$\phi_7 \in [0, 2\pi), \quad \Delta\phi_7 = \pi/16$$

$$\phi_{9,10} \in [0, 2\pi), \quad \Delta\phi_{9,10} = \pi/16$$

SM = green square

- $B \rightarrow X_s \bar{\ell} \ell$ Babar/Belle data: Br in q^2 -bin: $[1, 6]$ GeV 2
- $B \rightarrow K^* \bar{\ell} \ell$ Belle/CDF data: Br, A_{FB}, F_L in q^2 -bin: $[1, 6]$ GeV 2
 Br, A_{FB} in q^2 -bins: $[14.2, 16]$ GeV 2 and $[> 16]$ GeV 2

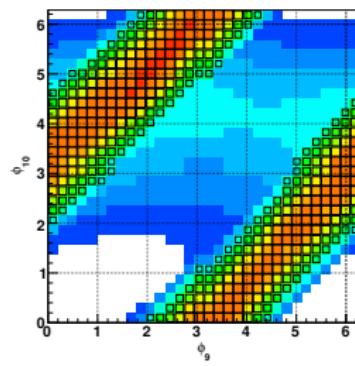
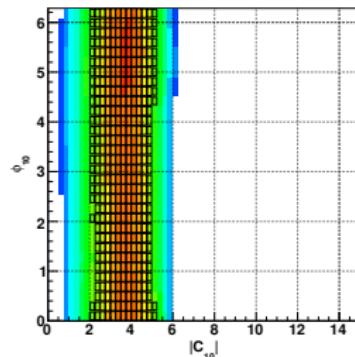
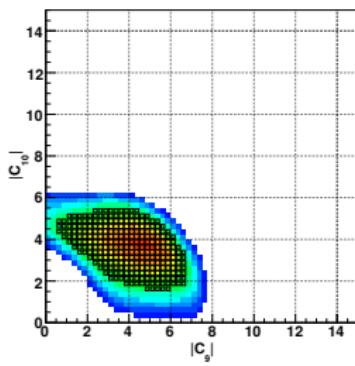
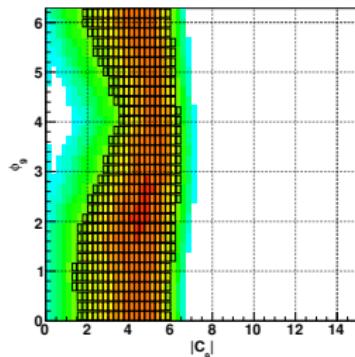
Determining 68 (95) % CL in 6D pmr-space $|C_{7,9,10}|$ and $\phi_{7,9,10} \rightarrow$ projection on $|C_9| - |C_{10}|$

⇒ without high- q^2 data [left] and with [right] → important impact,
 BUT form factors from lattice very desirable !!!

$$\Rightarrow Br(B_s \rightarrow \bar{\mu}\mu) < 1 \cdot 10^{-8} @ 95 \% \text{ CL}$$

FIT $C_{9,10}$ – COMPLEX – ONLY BELLE DATA

Model-indep. fit of complex $C_{9,10}$ ($C_9^{\text{SM}} = 4.2$, $C_{10}^{\text{SM}} = -4.2$)



$B \rightarrow K^* \bar{\ell} \ell$

- Br and A_{FB} in q^2 -bins

[1, 6] GeV^2
 [14.2, 16] GeV^2
 [> 16] GeV^2

- F_L in $q^2 \in [1, 6] \text{ GeV}^2$

$B \rightarrow X_s \bar{\ell} \ell$

- Br in [1, 6] GeV^2

$B \rightarrow K \bar{\ell} \ell$

- Br in [1, 6], [14.2, 16], [> 16] GeV^2

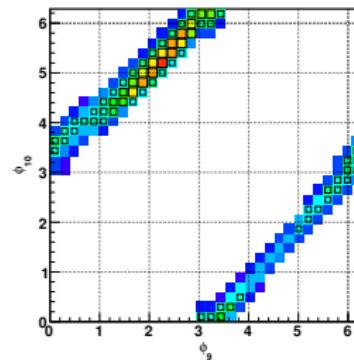
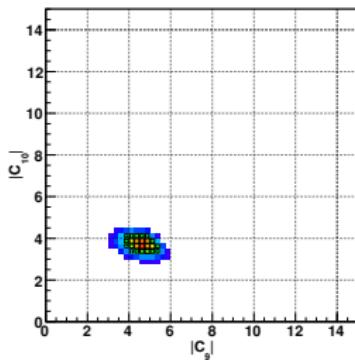
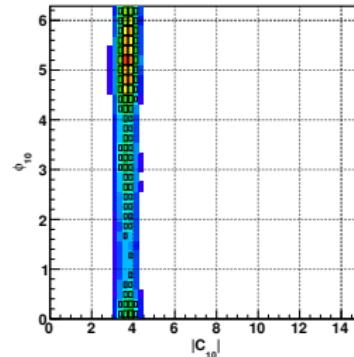
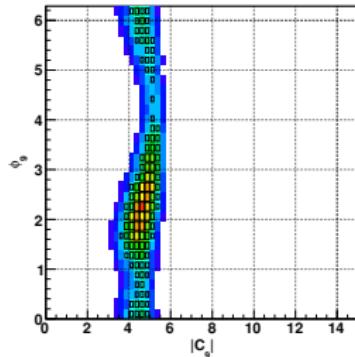
marginalised profile likelihood
 95 % (68 % box) CL regions

- $|C_7| = |C_7^{\text{SM}}|$
- $|C_{9,10}| \in [0, 15]$
- $\phi_{7,9,10} \in [0, 2\pi)$

preliminary
 Beaujean/CB/van Dyk/Wacker

Fit $C_{9,10}$ – COMPLEX – FUTURE?

For fun: keep exp. central values, divide all exp. errors by 5



$B \rightarrow K^* \bar{\ell} \ell$

- Br and A_{FB} in q^2 -bins

[1, 6] GeV^2

[14.2, 16] GeV^2

[> 16] GeV^2

- F_L in $q^2 \in [1, 6] \text{ GeV}^2$

$B \rightarrow X_s \bar{\ell} \ell$

- Br in [1, 6] GeV^2

$B \rightarrow K \bar{\ell} \ell$

- Br in [1, 6], [14.2, 16], [> 16] GeV^2

marginalised profile likelihood
95 % (68 % box) CL regions

- $|C_7| = |C_7^{\text{SM}}|$
- $|C_{9,10}| \in [0, 15]$
- $\phi_{7,9,10} \in [0, 2\pi)$

preliminary
Beaujean/CB/van Dyk/Wacker

CP-ASYMMETRIES @ HIGH- q^2

FF-FREE CP-ASYMMETRIES: SM OPERATOR BASIS

[CB/HILLER/VAN DYK ARXIV:1105.0376]

$$a_{\text{CP}}^{(1)} = \frac{\rho_1 - \bar{\rho}_1}{\rho_1 + \bar{\rho}_1}, \quad a_{\text{CP}}^{(2)} = \frac{\frac{\rho_2}{\rho_1} - \frac{\bar{\rho}_2}{\bar{\rho}_1}}{\frac{\rho_2}{\rho_1} + \frac{\bar{\rho}_2}{\bar{\rho}_1}}, \quad a_{\text{CP}}^{(3)} = 2 \frac{\rho_2 - \bar{\rho}_2}{\rho_1 + \bar{\rho}_1}$$

- NLO QCD corrections large
- still, theoretical uncertainties large: dominated by renorm. scale μ_b
- $a_{\text{CP}}^{\text{mix}}$ in $B_s \rightarrow \phi(\rightarrow K^+K^-) + \bar{\ell}\ell$

BSM OPERATORS @ HIGH- q^2 : SM' – I

Including BSM-operators: $\text{SM}' = O_{7', 9', 10'}$ [work in progress CB/Hiller/van Dyk]

$$A_{0,\parallel}^{L,R} = -C_-^{L,R} f_{0,\parallel}, \quad A_\perp^{L,R} = +C_+^{L,R} f_\perp$$

with universal coefficients $C^{L,R} \rightarrow C_\pm^{L,R}$

$$C_-^{L,R} = \left[(C_9^{\text{eff}} - C_{9'}^{\text{eff}}) + \kappa \frac{2m_b^2}{q^2} (C_7^{\text{eff}} - C_{7'}^{\text{eff}}) \right] \mp (C_{10} - C_{10'}),$$

$$C_+^{L,R} = \left[(C_9^{\text{eff}} + C_{9'}^{\text{eff}}) + \kappa \frac{2m_b^2}{q^2} (C_7^{\text{eff}} + C_{7'}^{\text{eff}}) \right] \mp (C_{10} + C_{10'})$$

Now the angular observables $I_i^{(k)}$ ($m_\ell = 0$) read

$$\frac{4}{3}(2I_2^s + I_3) = 2\rho_1^+ f_\perp^2, \quad \frac{4\sqrt{2}}{3}I_4 = 2\rho_1^- f_0 f_\parallel, \quad I_7 = 0,$$

$$\frac{4}{3}(2I_2^s - I_3) = 2\rho_1^- f_\parallel^2, \quad \frac{2\sqrt{2}}{3}I_5 = 4\text{Re}(\rho_2) f_0 f_\perp, \quad \frac{4\sqrt{2}}{3}I_8 = 4\text{Im}(\rho_2) f_0 f_\perp,$$

$$-\frac{4}{3}I_2^c = 2\rho_1^- f_0^2, \quad \frac{2}{3}I_6 = 4\text{Re}(\rho_2) f_\parallel f_\perp, \quad -\frac{4}{3}I_9 = 4\text{Im}(\rho_2) f_\parallel f_\perp$$

where ρ_1 and ρ_2 have to be generalised

$$\rho_1^\pm = \frac{1}{2} \left(|C_\pm^R|^2 + |C_\pm^L|^2 \right), \quad \rho_2 = \frac{1}{4} \left(C_+^R C_-^{R*} - C_-^L C_+^{L*} \right)$$

BSM OPERATORS @ HIGH- q^2 – II

Including BSM-operators [work in progress CB/Hiller/van Dyk]

- extension to $\rho_1 \rightarrow \rho_1^\pm$
- still have $H_T^{(1)} = 1$
- $I_7 = 0$, but $I_{8,9} \neq 0$
- generalisation: $H_T^{(2)} = H_T^{(3)} = 2 \frac{\text{Re}(\rho_2)}{\sqrt{\rho_1^- \cdot \rho_1^+}}$
- 2 new FF-free ratios

$$H_T^{(4)} = \frac{\sqrt{2} I_8}{\sqrt{-I_2^c (2I_2^s + I_3)}} = 2 \frac{\text{Im}(\rho_2)}{\sqrt{\rho_1^- \cdot \rho_1^+}}, \quad H_T^{(5)} = \frac{-I_9}{\sqrt{(2I_2^s)^2 - I_3^2}} = 2 \frac{\text{Im}(\rho_2)}{\sqrt{\rho_1^- \cdot \rho_1^+}}.$$

- $a_{\text{CP}}^{(1)} \rightarrow a_{\text{CP}}^{(1,\pm)}$ and $a_{\text{CP}}^{(2)} \rightarrow a_{\text{CP}}^{(2,\pm)}$
- generalisation of $a_{\text{CP}}^{(3)}$ and additional

$$a_{\text{CP}}^{(3)} = 2 \frac{\text{Re}(\rho_2 - \bar{\rho}_2)}{\sqrt{(\rho_1^+ - \bar{\rho}_1^+) \cdot (\rho_1^- - \bar{\rho}_1^-)}},$$

$$a_{\text{CP}}^{(4)} = 2 \frac{\text{Im}(\rho_2 - \bar{\rho}_2)}{\sqrt{(\rho_1^+ - \bar{\rho}_1^+) \cdot (\rho_1^- - \bar{\rho}_1^-)}}$$

- less “short-distance free” ratios, only $f_0/f_{||}$

$$B \rightarrow P + \bar{\ell}\ell$$

$B \rightarrow K + \bar{\ell}\ell$ @ HIGH- q^2 - I

FF RELATION (ISGUR/WISE) IN HEAVY QUARK LIMIT

$$f_T(q^2, \mu) = \frac{(M_B + M_K)M_B}{q^2} \kappa(\mu) f_+(q^2) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{M_B}\right)$$

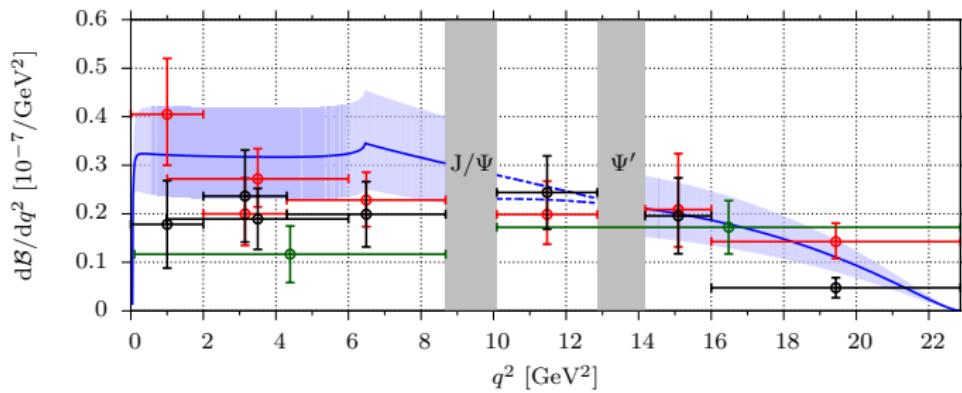
Grinstein/Pirjol hep-ph/0201298, hep-ph/0404250, $\kappa = 1 + \mathcal{O}(\alpha_s)$: known QCD matching correction

MATRIX ELEMENT @ HIGH- q^2

$$\mathcal{M}[\bar{B} \rightarrow \bar{K} \bar{\ell}\ell] \propto G_F \alpha_e V_{tb} V_{ts}^* f_+(q^2) \left(F_V p_B^\mu [\bar{\ell} \gamma_\mu \ell] + F_A p_B^\mu [\bar{\ell} \gamma_\mu \gamma_5 \ell] + F_P m_\ell [\bar{\ell} \gamma_5 \ell] \right)$$

$$F_A = \textcolor{red}{C_{10}}, \quad F_V = \textcolor{red}{C_9^{\text{eff}}} + \kappa \frac{2m_b^2}{q^2} \textcolor{red}{C_7^{\text{eff}}}, \quad F_P = \textcolor{red}{C_{10}} \left[\frac{(M_B^2 - M_K^2)}{q^2} \left(\frac{\textcolor{blue}{f}_0}{f_+} - 1 \right) - 1 \right]$$

$B \rightarrow K + \bar{\ell}\ell$ @ HIGH- q^2 - II



preliminary CB/Hiller/van Dyk/Wacker

for $\ell = e, \mu$ ($m_\ell = 0$)

$$\frac{d\Gamma}{dq^2} = \frac{\Gamma_0}{4} (\sqrt{\lambda})^3 \rho_1 f_+^2, \quad A_{CP}(q^2) = \frac{d\Gamma/dq^2 - d\bar{\Gamma}/dq^2}{d\Gamma/dq^2 + d\bar{\Gamma}/dq^2} = \frac{\rho_1 - \bar{\rho}_1}{\rho_1 + \bar{\rho}_1}$$

⇒ rate CP-asymmetry is also “long-distance-free” and

$$A_{CP}[B \rightarrow K\bar{\ell}\ell] = a_{CP}^{(1)}[B \rightarrow K^*\bar{\ell}\ell]$$

see also Bartsch/Beylich/Buchalla/Gao arXiv:0909.1512: $Br(B \rightarrow K + \bar{\nu}\nu)/Br(B \rightarrow K + \bar{\ell}\ell)$

CONCLUSION - I

- rich phenomenology in angular analysis of $B \rightarrow V_{on-shell} (\rightarrow P_1 P_2) + \bar{\ell}\ell$ to test flavour short-distance couplings – analogously $B_s \rightarrow \phi (\rightarrow K^+ K^-) + \bar{\ell}\ell$
- low- q^2 and high- q^2 regions in $b \rightarrow s + \bar{\ell}\ell$ accessible via power exp's (QCDF, SCET, OPE + HQET) → reveal symmetries of QCD dynamics
- reducing Non-PT uncertainties by suitable ratios of observables guided by power exp's → allowing for quite precise theory predictions for exclusive decays
- low- q^2 theoretically well understood (even $(\bar{c}c)$ -resonances can be estimated)
→ many interesting tests, waiting for data
- high- q^2 :
 - $(\bar{c}c)$ -resonances seem under control, violation of $H_T^{(1)} = 1$ can be tested
 - “long-distance free” ratios $H_T^{(2,3)}$ to test SM
 - “short-distance free” ratios to test q^2 -dep. of FF-ratios directly with lattice
 - need FF input from Lattice → required to exploit exp. data dBr/dq^2

CONCLUSION – II

Other phenomenological topics

- separate measurement of $\ell = e$ and $\ell = \mu$: investigate ratios of $I_i^{(k)}(\ell = e)/I_i^{(k)}(\ell = \mu)$ in analogy to R_{K^*} @ low- and high- q^2 → ℓ -flavour non-universal effects
- combination of $B \rightarrow K\bar{\ell}\ell$ and $B \rightarrow K^*\bar{\ell}\ell$ – work in progress CB/Hiller/van Dyk/Wacker
- measurement of $B \rightarrow (K, K^*) + \bar{\tau}\tau$ feasible ???
→ interesting for BSM scenarios with scalar and pseudo-scalar operators
- combined measurement of $B \rightarrow K + \bar{\nu}\nu$ and $B \rightarrow K + \bar{\ell}\ell$ – Bartsch/Beylich/Buchalla/Gao
arXiv:09091512
- $B \rightarrow X_s \bar{\ell}\ell$ @ high- q^2

EOS = new Flavour tool @ TU Dortmund by Danny van Dyk et al.

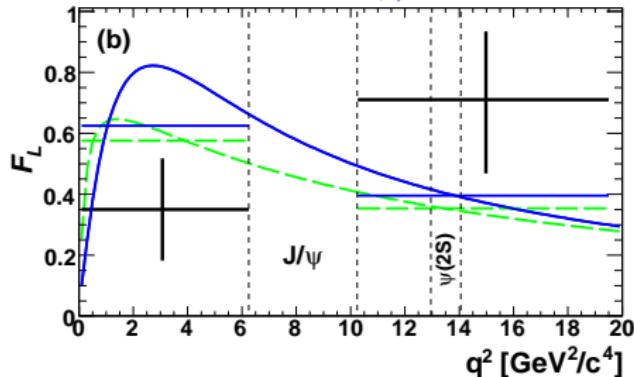
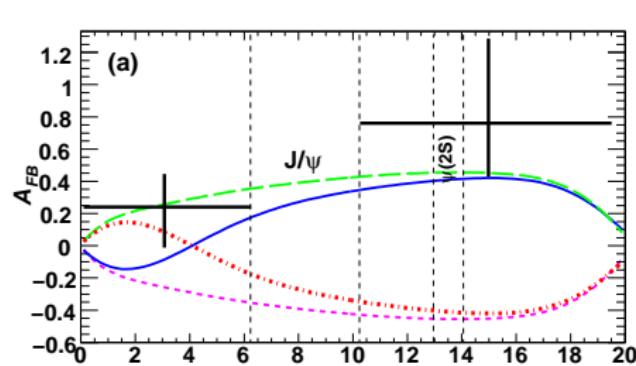
<http://project.het.physik.tu-dortmund.de/eos/>

⇒ Danny van Dyk presentation tomorrow

Backup slides

BABAR [ARXIV:0804.4412]

Analysis of 384 M $B\bar{B}$ pairs → search all channels $B^{+,0}$, $K^{(*),+,-}$ and $\ell = e, \mu$



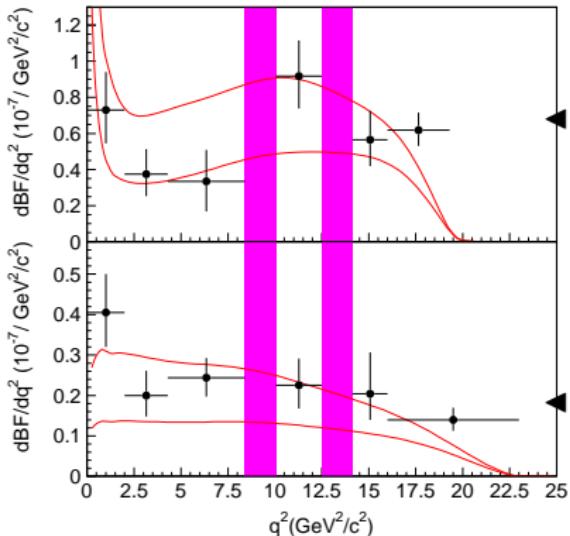
- 2 bins: low- $q^2 \in [0.1 - 6.25] \text{ GeV}^2$ and high- $q^2 > 10.24 \text{ GeV}^2$
 $\Rightarrow (27 \pm 6) + (37 \pm 10) = 64 \text{ events}$
- veto of J/ψ and ψ' regions: background $B \rightarrow K^*(\bar{c}c) \rightarrow K^*\ell\bar{\ell}$
- angular analysis in each q^2 -bin in θ_ℓ and θ_{K^*} ⇒ fit F_L and A_{FB}

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_{K^*}} = \frac{3}{2} F_L \cos^2 \theta_{K^*} + (1 - F_L)(1 - \cos^2 \theta_{K^*}),$$

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_\ell} = \frac{3}{4} F_L (1 - \cos^2 \theta_\ell) + \frac{3}{8} (1 - F_L)(1 + \cos^2 \theta_\ell) + A_{FB} \cos \theta_\ell$$

BELLE [ARXIV:0904.0770]

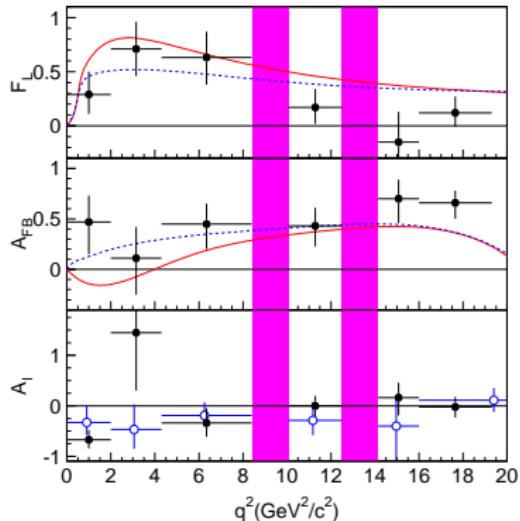
Analysis of 657 M $B\bar{B}$ pairs = 605 fb^{-1} → search all channels $B^{+,0}$, $K^{(*),+,-}$ and $\ell = e, \mu$



$\blacktriangleleft B \rightarrow K^* \ell \bar{\ell}$

red = SM

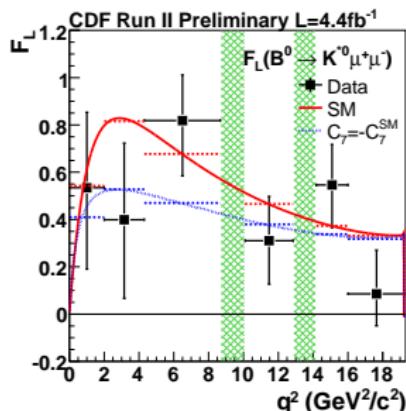
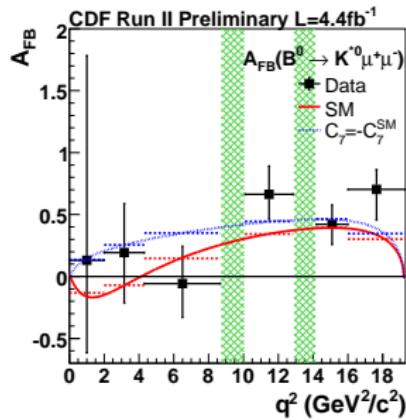
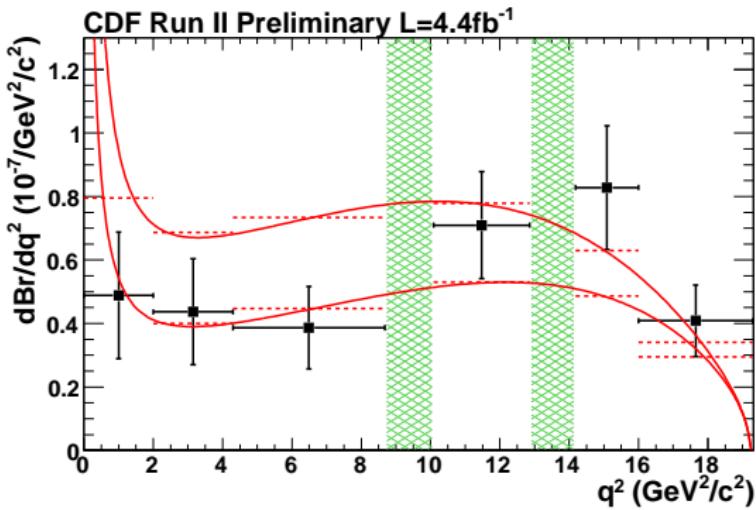
$\blacktriangleleft B \rightarrow K \ell \bar{\ell}$



- 6 bins ⇒ 247 events (121 @ $q^2 > 14 \text{ GeV}^2$)
- angular analysis in each q^2 -bin in θ_ℓ and θ_{K^*} ⇒ fit F_L and A_{FB}
- all- q^2 extrapolated results:
 - $Br = (10.7^{+1.1}_{-1.0} \pm 0.09) \times 10^{-7}$, $A_{CP} = -0.10 \pm 0.10 \pm 0.01$,
 - $R_{K^*} = 0.83 \pm 0.17 \pm 0.08$ (SM = 0.75), $A_I = -0.29^{+0.16}_{-0.16} \pm 0.09$ ($q^2 < 8.68 \text{ GeV}^2$)

CDF [ARXIV:1101.1028]

- analysis of 4.4 fb^{-1} (CDF Run II) \Rightarrow only $B^0 \rightarrow K^{*0} \bar{\mu}\mu$
- discovery of $B_s \rightarrow \phi \bar{\mu}\mu$ 6.3σ (27 ± 6) events
- 101 events (42 @ $q^2 > 14 \text{ GeV}^2$) - Belle q^2 -binning



EXPERIMENTAL PROSPECTS – I

Improvement of current experiments

BABAR + BELLE analysis of final data set in progress

CDF $(2 - 3) \times$ CDF data set through 2011 from $4.4 \text{ fb}^{-1} \rightarrow (9 - 13) \text{ fb}^{-1}$
(more data, improved analysis and final states)

LHCb prospects for 2.0 fb^{-1} (= by the end of 2012 ???)

$\ell = \mu$ expected events after selection [arXiv:0912.4179]

a) cut-based: (4200^{+1100}_{-1000}) events

$(B/S = 0.05 \pm 0.04 \text{ and } S/\sqrt{S+B} = 63^{+9}_{-8})$

b) multivariate: (6200^{+1700}_{-1500}) events

$(B/S = 0.25 \pm 0.08 \text{ and } S/\sqrt{S+B} = 71^{+11}_{-10})$

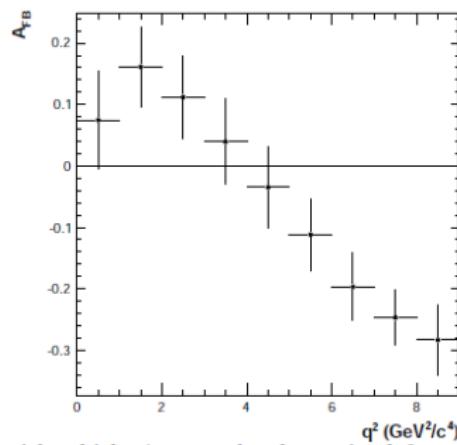
→ currently: (Babar + Belle + CDF) ≈ 410 events

q^2_0 of A_{FB} expected with stat. unc. of $\pm 0.5 \text{ GeV}^2$

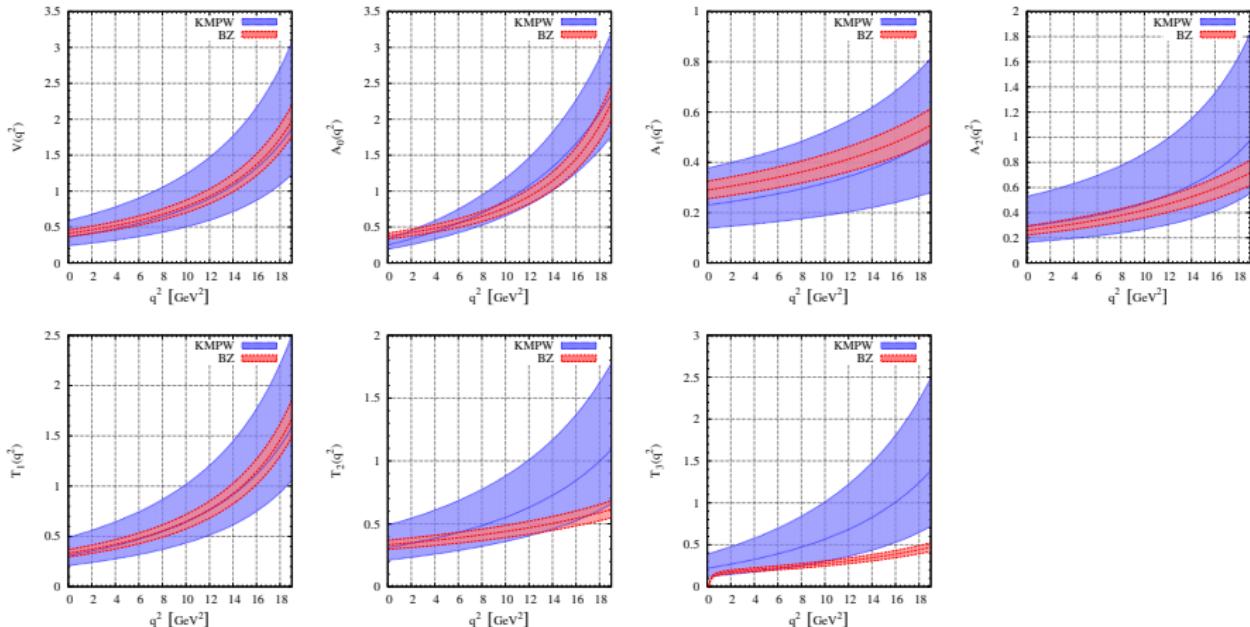
(B factory uncertainty expected with 0.3 fb^{-1})

$\ell = e$ $\sim (200 - 250)$ events per 2.0 fb^{-1} with $S/B \sim 1$

[LHCb-PUB-2009-008]



EXTRAPOLATION OF LCSR $B \rightarrow K^*$ FF's TO HIGH- q^2



BZ = Ball/Zwicky hep-ph/0412079

KMPW = Khodjamirian/Mannel/Pivovarov/Wang arXiv:1006.4945

numerically most important for $B \rightarrow K^* \bar{\ell} \ell$: V and A_1