Angular Analysis of the Decay $\Lambda_b \to \Lambda[\to N\pi]\ell^+\ell^-$

Philipp Böer, Thorsten Feldmann, and • Danny van Dyk

Naturwissenschaftlich-Technische Fakultät - Theoretische Physik 1
Universität Siegen

4th Workshop on flavour symmetries and consequences in accelerators and cosmology, Brighton
17.6.2014

Bundesministerium für Bildung und Forschung

UNIVERSITÄT SIEGEN

DFG FOR 1873
quark **f**lavour physics and **e**ffective field **t**heories

Effective Field Theory

Flavor Changing Neutral Current (FCNC)

- at parton level: $\Lambda_b \hat{=} (u\, d\, b) \to \Lambda \hat{=} (u\, d\, s)$
- handle in effective theory (all fields heavier than b are integrated out)
  - matrix elements of operators $\mathcal{O}_i$ represent physics below $\mu \simeq m_b$
  - Wilson coefficients $\mathcal{C}_i \equiv \mathcal{C}_i(M_W, M_Z, m_t, \dots)$ represent physics above $\mu \simeq m_b$

Effective Hamiltonian

$$\mathcal{H} = -\frac{4G_\mathrm{F}}{\sqrt{2}}\left[V_{tb}V_{ts}^* \sum_i \mathcal{C}_i \mathcal{O}_i + O\left(V_{ub}V_{us}^*\right)\right] + \text{h.c.}$$

Effective Field Theory

Flavor Changing Neutral Current (FCNC)

- at parton level: $\Lambda_b \hat{=} (u\,d\,b) \to \Lambda \hat{=} (u\,d\,s)$
- handle in effective theory (all fields heavier than b are integrated out)
  - matrix elements of operators $\mathcal{O}_i$ represent physics below $\mu \simeq m_b$
  - Wilson coefficients $\mathcal{C}_i \equiv \mathcal{C}_i(M_W, M_Z, m_t, \dots)$ represent physics above $\mu \simeq m_b$

Decays

$$B \to K^* \ell^+ \ell^- \quad B_s \to \mu^+ \mu^- \quad B \to K \ell^+ \ell^-$$

$$B \to K^* \gamma \quad B \to X_s \ell^+ \ell^- \quad B \to X_s \gamma$$

$$\Lambda_b \to \Lambda[\to N\pi]\ell^+ \ell^-$$

Why $\Lambda_b \to \Lambda$

$B \to K^* \ell^+ \ell^-$ is being measured with increasing precision. Why spend effort on $\Lambda_b \to \Lambda$?

pro arguments, sorted from weakest to strongest

Why $\Lambda_b \to \Lambda$

$B \to K^* \ell^+ \ell^-$ is being measured with increasing precision. Why spend effort on $\Lambda_b \to \Lambda$?

pro arguments, sorted from weakest to strongest

- independent confirmation of results: same $b \to s\ell^+\ell^-$ operators, different hadronic matrix elements

Why $\Lambda_b \to \Lambda$

$B \to K^* \ell^+ \ell^-$ is being measured with increasing precision. Why spend effort on $\Lambda_b \to \Lambda$?

pro arguments, sorted from weakest to strongest

- independent confirmation of results: same $b \to s\ell^+\ell^-$ operators, different hadronic matrix elements
- $\Gamma_\Lambda \simeq 2.5 \cdot 10^{-6}$ eV: small width approximation fully applicable (compare $B \to K\pi\ell^+\ell^-$ where non-res. P-wave contributions are unconstrained)

Why $\Lambda_b \to \Lambda$

$B \to K^* \ell^+ \ell^-$ is being measured with increasing precision. Why spend effort on $\Lambda_b \to \Lambda$?

pro arguments, sorted from weakest to strongest

- independent confirmation of results: same $b \to s\ell^+\ell^-$ operators, different hadronic matrix elements
- $\Gamma_\Lambda \simeq 2.5 \cdot 10^{-6}$ eV: small width approximation fully applicable (compare $B \to K\pi\ell^+\ell^-$ where non-res. P-wave contributions are unconstrained)
- fewer hadronic matrix elements (2 in HQET limit, 1 in SCET limit)

Why $\Lambda_b \to \Lambda$

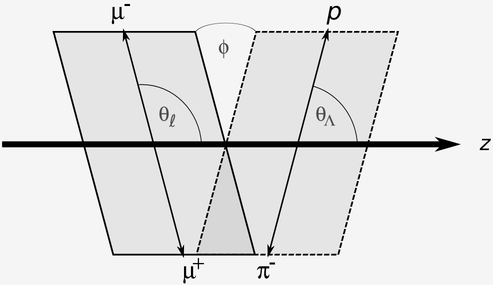
$B \to K^*\ell^+\ell^-$ is being measured with increasing precision. Why spend effort on $\Lambda_b \to \Lambda$?

pro arguments, sorted from weakest to strongest

- independent confirmation of results: same $b \to s\ell^+\ell^-$ operators, different hadronic matrix elements
- $\Gamma_\Lambda \simeq 2.5 \cdot 10^{-6}$ eV: small width approximation fully applicable (compare $B \to K\pi\ell^+\ell^-$ where non-res. P-wave contributions are unconstrained)
- fewer hadronic matrix elements (2 in HQET limit, 1 in SCET limit)
- doubly weak decay: complementary constraints on $b \to s\ell^+\ell^-$ physics with respect to $B \to K^*\ell^+\ell^-$

Kinematics and Decay Topology

$$\Lambda_b(p) \to \Lambda(k)\,[\to N(k_1)\,\pi(k_2)]\,\ell^+(q_1)\,\ell^-(q_2)$$



3 independent decay angles

- $\cos\theta_\Lambda \sim \bar{k}\cdot q$
- $\cos\theta_\ell \sim k\cdot\bar{q}$
- $\cos\phi \sim \bar{k}\cdot\bar{q}$

only for unpolarized Λ_b

Momenta

$$q = q_1 + q_2$$
$$\bar{q} = q_1 - q_2$$
$$k = k_1 + k_2$$
$$\bar{k} = k_1 - k_2$$

$\Lambda_b \to \Lambda$ Hadronic Matrix Elements

In General

- using (overall 10) helicity form factors (FFs) [compare Feldmann/Yip 1111.1844]

$$\varepsilon^\dagger(\lambda) \cdot \langle \Lambda | \Gamma | \Lambda_b \rangle \sim f_\lambda^\Gamma(q^2)$$

- for lepton mass $m_\ell \to 0$ this reduces to 8 independent FFs
- results for all FFs expected from the lattice in the long run

Within HQET

- heavy quark spin symmmetry reduces matrix elements to 2 FFs at leading power
- known from Lattice QCD [Detmold/Lin/Meinel/Wingate 1212.4827]

Within SCET applicable when $E_\Lambda = O(m_b)$

- matrix elements reduce further to 1 single FF
- estimates from SCET sum rules [Feldmann/Yip 1111.1844]

$\Lambda \to N\pi$ Hadronic Matrix Element

- $\Lambda \to N\pi$ is a parity-violating weak decay
- branching fraction $\mathcal{B}[\Lambda \to N\pi] = (99.7 \pm 0.1)\%$ [PDG, our naive average]

- equations of motions reduce independent matrix elements to 2
- we choose to express them through
  - decay width Γ_Λ
  - parity-violating coupling α

- within small width approximation, Γ_Λ cancels
- α well known from experiment: $\alpha_{p\pi^-} = 0.642 \pm 0.013$ [PDG average]

Operator Basis

Semileptonic/Radiative Operators

SM and chirality-flipped operators

$$\mathcal{O}_{9(9')} = \frac{\alpha_e}{4\pi}[\bar{s}\gamma^\mu P_{L(R)} b][\bar{\ell}\gamma_\mu \ell] \qquad \mathcal{O}_{10(10')} = \frac{\alpha_e}{4\pi}[\bar{s}\gamma^\mu P_{L(R)} b][\bar{\ell}\gamma_\mu \gamma_5 \ell]$$

$$\mathcal{O}_{7(7')} = \frac{e\overline{m_b}}{4\pi}[\bar{s}\sigma^{\mu\nu} P_{R(L)} b] F_{\mu\nu}$$

Hadronic Operators

contribute via intermediate off-shell photon

$$\mathcal{O}_1 = [\bar{c}\gamma^\mu P_L T^A b][\bar{s}\gamma_\mu P_L T^A c]$$

$$\mathcal{O}_2 = [\bar{c}\gamma^\mu P_L b][\bar{s}\gamma_\mu P_L c]$$

Hadronic Contributions

all exclusive $b \to s\ell^+\ell^-$ processes face problem of hadronic contributions

- hadronic operators give rise to e.g. $b \to s\bar{c}c$, hadronizes to $\Lambda_b \to \Lambda J/\psi(\to \ell^+\ell^-)$
- systematically include effects via hadronic two-point function $\mathcal{T}(q^2)$

$$C_7\langle\mathcal{O}_7\rangle \to \mathcal{T}(q^2)$$

- different approaches to obtain $\mathcal{T}(q^2)$, depending on kinematics
- $B \to K^*\ell^+\ell^-$: methods and domain of validity
  - small $q^2 \ll m_b^2$: QCD Factorization (QCDF) [Beneke/Feldmann/Seidel hep-ph/0106067 and hep-ph/0412400]
  - large $q^2 \simeq m_b^2$: Operator Product Expansion (OPE) [Grinstein/Pirjol hep-ph/0404250]
- $\Lambda_b \to \Lambda\ell^+\ell^-$: the low recoil OPE can be directly applied at $q^2 \simeq m_b^2$

Angular Distribution of $\Lambda_b \to \Lambda[\to N\pi]\ell^+\ell^-$

we define the angular distribution as

$$\frac{8\pi}{3}\frac{\mathrm{d}^4\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_\Lambda\,\mathrm{d}\phi} \equiv K(q^2, \cos\theta_\ell, \cos\theta_\Lambda, \phi)$$

when considering only SM and chirality-flipped operators

$$\begin{aligned} K = 1 & \left(K_{1ss}\sin^2\theta_\ell + K_{1cc}\cos^2\theta_\ell + K_{1c}\cos\theta_\ell\right) \\ + \cos\theta_\Lambda & \left(K_{2ss}\sin^2\theta_\ell + K_{2cc}\cos^2\theta_\ell + K_{2c}\cos\theta_\ell\right) \\ + \sin\theta_\Lambda\sin\phi & \left(K_{3sc}\sin\theta_\ell\cos\theta_\ell + K_{3s}\sin\theta_\ell\right) \\ + \sin\theta_\Lambda\cos\phi & \left(K_{4sc}\sin\theta_\ell\cos\theta_\ell + K_{4s}\sin\theta_\ell\right) \end{aligned}$$

$$K_n \equiv K_n(q^2)$$

Angular Observables

- matrix elements parametrized through 8 transversity amplitudes $A^{\lambda}_{\chi_M}$

$$A^R_{\perp_1}\,,\ A^R_{\|_1}\,,\ A^R_{\perp_0}\,,\ A^R_{\|_0}\,,\ \text{and } (R \leftrightarrow L)$$

λ dilepton chirality

χ transversity state, similar as in $B \to K^* \ell^+ \ell^-$

M |third component| of dilepton angular momentum

Angular Observables

- matrix elements parametrized through 8 transversity amplitudes $A^{\lambda}_{\chi_M}$

$$A^R_{\perp_1}\,,\; A^R_{\|_1}\,,\; A^R_{\perp_0}\,,\; A^R_{\|_0}\,,\; \text{and } (R \leftrightarrow L)$$

  - λ dilepton chirality
  - χ transversity state, similar as in $B \to K^* \ell^+ \ell^-$
  - M |third component| of dilepton angular momentum

- express angular observables through transversity amplitudes, e.g.

$$K_{1cc} = \frac{1}{2}\left[|A^R_{\perp_1}|^2 + |A^R_{\|_1}|^2 + (R \leftrightarrow L)\right]$$
$$K_{2c} = \frac{\alpha}{2}\left[|A^R_{\perp_1}|^2 + |A^R_{\|_1}|^2 - (R \leftrightarrow L)\right]$$
$$\vdots$$

full list in the backups

Simple Observables

start with integrated decay width

$$\Gamma = 2K_{1ss} + K_{1cc}$$

define further observables X as weighted (ω_X) integrals

$$X = \frac{1}{\Gamma}\int \frac{\mathrm{d}^3\Gamma}{\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_\Lambda\,\mathrm{d}\phi}\omega_X(\cos\theta_\ell, \cos\theta_\Lambda, \phi)\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_\Lambda\,\mathrm{d}\phi$$

A leptonic forward-backward asymmetry

$$A^\ell_{\mathrm{FB}} = \frac{3}{2}\frac{K_{1c}}{2K_{1ss} + K_{1cc}} \qquad \text{with } \omega_{A^\ell_{\mathrm{FB}}} = \mathrm{sgn}\cos\theta_\ell$$

Simple Observables

start with integrated decay width

$$\Gamma = 2K_{1ss} + K_{1cc}$$

define further observables X as weighted (ω_X) integrals

$$X = \frac{1}{\Gamma}\int \frac{\mathrm{d}^3\Gamma}{\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_\Lambda\,\mathrm{d}\phi}\omega_X(\cos\theta_\ell, \cos\theta_\Lambda, \phi)\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_\Lambda\,\mathrm{d}\phi$$

A leptonic forward-backward asymmetry

$$A_{\mathrm{FB}}^\ell = \frac{3}{2}\frac{K_{1c}}{2K_{1ss} + K_{1cc}} \qquad \text{with } \omega_{A_{\mathrm{FB}}^\ell} = \operatorname{sgn}\cos\theta_\ell$$

B fraction of longitudinal dilepton pairs

$$F_0 = \frac{2K_{1ss} - K_{1cc}}{2K_{1ss} + K_{1cc}} \qquad \text{with } \omega_{F_0} = 2 - 5\cos^2\theta_\ell$$

Simple Observables

start with integrated decay width

$$\Gamma = 2K_{1ss} + K_{1cc}$$

define further observables X as weighted (ω_X) integrals

$$X = \frac{1}{\Gamma} \int \frac{\mathrm{d}^3\Gamma}{\mathrm{d}\cos\theta_\ell \, \mathrm{d}\cos\theta_\Lambda \, \mathrm{d}\phi} \omega_X(\cos\theta_\ell, \cos\theta_\Lambda, \phi) \mathrm{d}\cos\theta_\ell \, \mathrm{d}\cos\theta_\Lambda \, \mathrm{d}\phi$$

C hadronic forward-backward asymmetry

$$A_{\mathrm{FB}}^{\Lambda} = \frac{1}{2} \frac{2K_{2ss} + K_{2cc}}{2K_{1ss} + K_{1cc}} \qquad \text{with } \omega_{A_{\mathrm{FB}}^{\ell}} = \mathrm{sgn}\cos\theta_\Lambda$$

Simple Observables

start with integrated decay width

$$\Gamma = 2K_{1ss} + K_{1cc}$$

define further observables X as weighted (ω_X) integrals

$$X = \frac{1}{\Gamma}\int \frac{\mathrm{d}^3\Gamma}{\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_\Lambda\,\mathrm{d}\phi}\omega_X(\cos\theta_\ell,\cos\theta_\Lambda,\phi)\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_\Lambda\,\mathrm{d}\phi$$

C hadronic forward-backward asymmetry

$$A_{\mathrm{FB}}^{\Lambda} = \frac{1}{2}\frac{2K_{2ss}+K_{2cc}}{2K_{1ss}+K_{1cc}} \qquad \text{with } \omega_{A_{\mathrm{FB}}^{\ell}} = \operatorname{sgn}\cos\theta_\Lambda$$

D combined forward-backward asymmetry

$$A_{\mathrm{FB}}^{\ell\Lambda} = \frac{3}{4}\frac{K_{2c}}{2K_{1ss}+K_{1cc}} \qquad \text{with } \omega_{A_{\mathrm{FB}}^{\ell}} = \operatorname{sgn}\cos\theta_\Lambda\ \operatorname{sgn}\cos\theta_\ell$$

- ingredients
  - Low Recoil OPE expected valid only for q^2-integrated quantities
    [Grinstein/Pirjol hep-ph/0404250], [Beylich/Buchalla/Feldmann 1101.5118]
  - to leading power in the $1/m_b$ expansion there are only two independent form factors

$$\langle\Lambda|\Gamma|\Lambda_b\rangle \sim \bar{u}_\Lambda\left[\xi_1\Gamma + \xi_2 \not{v}\Gamma\right] u_{\Lambda_b} \qquad \text{with } v\text{: } \Lambda_b \text{ velocity}$$

At Low Recoil: Amplitudes when $q^2 = O\left(m_b^2\right)$

- ingredients
  - Low Recoil OPE expected valid only for q^2-integrated quantities
    [Grinstein/Pirjol hep-ph/0404250], [Beylich/Buchalla/Feldmann 1101.5118]
  - to leading power in the $1/m_b$ expansion there are only two independent form factors

$$\langle\Lambda|\Gamma|\Lambda_b\rangle \sim \bar{u}_\Lambda\left[\xi_1\Gamma + \xi_2 \not{v}\Gamma\right]u_{\Lambda_b} \qquad \text{with } v\text{: } \Lambda_b \text{ velocity}$$

- all amplitudes factorize
  - schematically — see backups for full expr

$$A^{R(L)} \propto C_\pm^{R(L)} \times f + \text{corrections}$$

  - for only SM-like operators coefficients unify $C_\pm^{R(L)} \to C^{R(L)}$
  - corrections suppressed: $O\left(\alpha_s \frac{\Lambda}{m_b}\right)$ and $O\left(\frac{C_7}{C_9}\frac{\Lambda}{m_b}\right)$

At Low Recoil: Observables when $q^2 = O\left(m_b^2\right)$

- observables are bilinears of $C_\pm^{R(L)}$
  - some are the same as in $B \to K^* \ell^+ \ell^-$ [Bobeth/Hiller/DvD 1006.5013 and 1212.2321]

$$\rho_1^\pm = \frac{1}{2}\left(|C_\pm^R|^2 + |C_\pm^L|^2\right) \qquad \rho_2 = \frac{1}{4}\left(C_+^R C_-^{R*} - C_-^L C_+^{L*}\right)$$

  - new complementary combinations (also in $B \to K\pi\ell^+\ell^-$, see talk by Gudrun Hiller)

$$\rho_3^\pm = \frac{1}{2}\left(|C_\pm^R|^2 - |C_\pm^L|^2\right) \qquad \rho_4 = \frac{1}{4}\left(C_+^R C_-^{R*} + C_-^L C_+^{L*}\right)$$

- observables are bilinears of $C_\pm^{R(L)}$
  - some are the same as in $B \to K^* \ell^+ \ell^-$ [Bobeth/Hiller/DvD 1006.5013 and 1212.2321]

$$\rho_1^\pm = \frac{1}{2}\left(|C_\pm^R|^2 + |C_\pm^L|^2\right) \qquad \rho_2 = \frac{1}{4}\left(C_+^R C_-^{R*} - C_-^L C_+^{L*}\right)$$

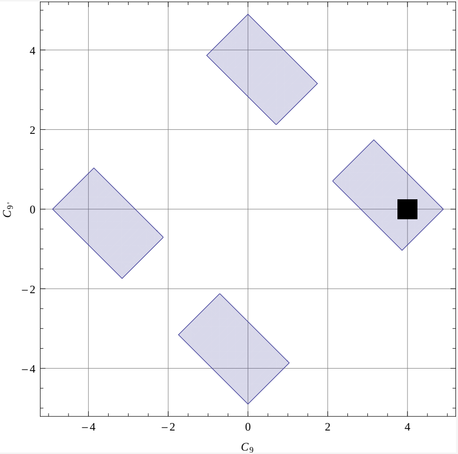
  - new complementary combinations (also in $B \to K\pi\ell^+\ell^-$, see talk by Gudrun Hiller)

$$\rho_3^\pm = \frac{1}{2}\left(|C_\pm^R|^2 - |C_\pm^L|^2\right) \qquad \rho_4 = \frac{1}{4}\left(C_+^R C_-^{R*} + C_-^L C_+^{L*}\right)$$

- which observables probe these new combinations?
  - hadronic forward-backward asymmetry $A_{\rm FB}^{\Lambda}$ is sensitive to $\mathrm{Re}\left(\rho_3^\pm\right)$
  - combined forward-backward asymmetry $A_{\rm FB}^{\ell\Lambda}$ is sensitive to $\mathrm{Re}\left(\rho_4\right)$

Sketch of Hypothetical Constraints in NP Model

assume $\mathcal{C}_{9(9')}$ free floating, and $\mathcal{C}_{10(10')} = \mathcal{C}^{\mathrm{SM}}_{10(10')} \simeq (-4, 0)$

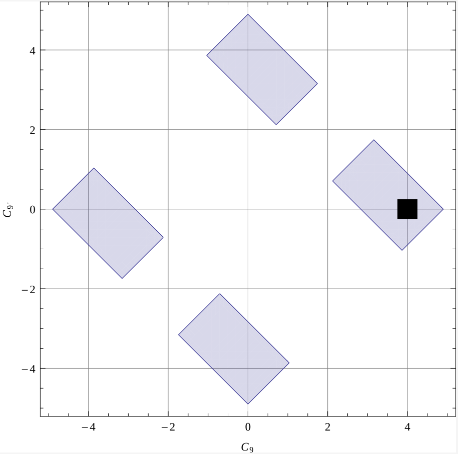


- existing constraints
  - $\rho_1^\pm$ blue banded constraints

black square: SM point

Sketch of Hypothetical Constraints in NP Model

assume $\mathcal{C}_{9(9')}$ free floating, and $\mathcal{C}_{10(10')} = \mathcal{C}^{\text{SM}}_{10(10')} \simeq (-4, 0)$



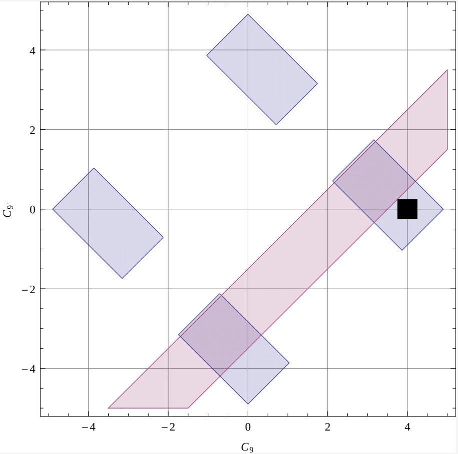
- existing constraints

  $\rho_1^{\pm}$ blue banded constraints
  ρ_2 insensitive (not shown)

black square: SM point

Sketch of Hypothetical Constraints in NP Model

assume $\mathcal{C}_{9(9')}$ free floating, and $\mathcal{C}_{10(10')} = \mathcal{C}^{\mathrm{SM}}_{10(10')} \simeq (-4, 0)$



- existing constraints

  $\rho_1^{\pm}$ blue banded constraints

  ρ_2 insensitive (not shown)

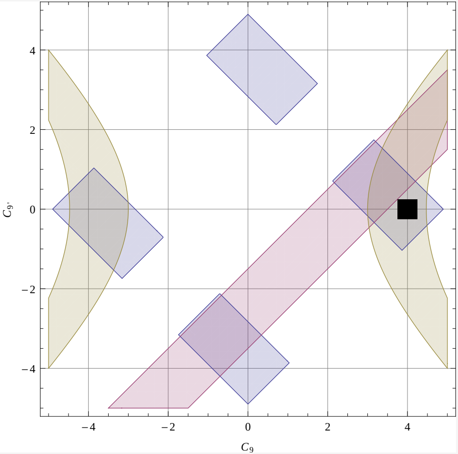
- new constraints

  ρ_3^{-} purple banded constraints

black square: SM point

Sketch of Hypothetical Constraints in NP Model

assume $\mathcal{C}_{9(9')}$ free floating, and $\mathcal{C}_{10(10')} = \mathcal{C}^{\text{SM}}_{10(10')} \simeq (-4, 0)$



- existing constraints

  $\rho_1^\pm$ blue banded constraints
  ρ_2 insensitive (not shown)

- new constraints

  ρ_3^- purple banded constraints
  ρ_4 gold hyperbolic constraint

black square: SM point

Optimized Observables

Goals

construct observables . . .

- . . . that predominantly test short-distance physics
- . . . that probe ratios of form factors

Short-distance sensitive observables

- in SM + SM' basis

$$\frac{K_{2cc}}{K_{1c}} = \frac{\alpha\,\mathrm{Re}\,(\rho_4)}{\mathrm{Re}\,(\rho_2)}$$

- in SM basis

$$\frac{K_{2cc}}{K_{1c}} = \frac{\alpha\rho_1}{2\,\mathrm{Re}\,(\rho_2)} \qquad \frac{K_{1cc}}{K_{2c}} = \frac{2\alpha\,\mathrm{Re}\,(\rho_2)}{\rho_1}$$

Form-factor ratios

- in SM + SM' only two ratios can be probed

$$\frac{2K_{2ss}}{K_{2cc}} \sim 1 + \frac{f_0^V f_0^A}{f_\perp^V f_\perp^A} \qquad \frac{2K_{4sc}}{K_{2cc}} \sim \frac{f_0^V}{f_\perp^V} - \frac{f_0^A}{f_\perp^A}$$

- in SM basis also f_0^V/f_0^A, $f_\perp^V/f_\perp^A$ can be probed

Numeric Results

- preliminary results
  - low recoil region $14.18\text{GeV}^2 \leq q^2 \leq 20.30\text{GeV}^2$
  - scale $\mu = 4.2\text{GeV}$
  - uncertainties: form factors, CKM, quark masses

$$
\begin{aligned}
\langle \mathcal{B} \rangle &= (5.2 \pm 0.8) \cdot 10^{-7} \\
\langle F_0 \rangle &= 0.46 \pm 0.2 \\
\langle A_{\text{FB}}^{\ell} \rangle &= -0.266^{+0.012}_{-0.015} \\
\langle A_{\text{FB}}^{\Lambda} \rangle &= +0.248^{+0.006}_{-0.007} \\
\langle A_{\text{FB}}^{\ell\Lambda} \rangle &= -0.127 \pm 0.005
\end{aligned}
$$

- agrees with $\mathcal{B}$ measurement [LHCb 1306.2577, our naive average]

$$\langle \mathcal{B} \rangle = (6.2 \pm 2.1) \cdot 10^{-7}$$

Conclusion

- rare decay $\Lambda_b \to \Lambda(\to N\pi)\ell^+\ell^-$ mediated by $b \to s\ell^+\ell^-$
  - self-analyzing through weak $\Lambda \to N\pi$ decay
  - offers complementary new constraints on $b \to s\ell^+\ell^-$ physics
- decay induces angular distribution with 10 (or more) angular observables
  - three forward-backward asymmetries, two of which probe new constraints: $A_{\mathrm{FB}}^{\Lambda}$ and $A_{\mathrm{FB}}^{\ell\Lambda}$
- predictions for observables at low recoil using HQET form factor relations

Outlook

- at low q^2: spectator interaction basically unknown
- wait for lattice results on all 10 form factors for $\Lambda_b \to \Lambda$

Backup Slides

Angular Observables

$$K_{1ss} = \frac{1}{4}\left[|A^R_{\perp_1}|^2 + |A^R_{\parallel_1}|^2 + 2|A^R_{\perp_0}|^2 + 2|A^R_{\parallel_0}|^2 + (R \leftrightarrow L)\right]$$

$$K_{1cc} = \frac{1}{2}\left[|A^R_{\perp_1}|^2 + |A^R_{\parallel_1}|^2 + (R \leftrightarrow L)\right]$$

$$K_{1c} = -\operatorname{Re}\left(A^R_{\perp_1} A^{*R}_{\parallel_1} - (R \leftrightarrow L)\right)$$

$$K_{2ss} = -\frac{\alpha}{2}\operatorname{Re}\left(A^R_{\perp_1} A^{*R}_{\parallel_1} + 2A^R_{\perp_0} A^{*R}_{\parallel_0} + (R \leftrightarrow L)\right)$$

$$K_{2cc} = -\alpha\operatorname{Re}\left(A^R_{\perp_1} A^{*R}_{\parallel_1} + (R \leftrightarrow L)\right)$$

$$K_{2c} = \frac{\alpha}{2}\left[|A^R_{\perp_1}|^2 + |A^R_{\parallel_1}|^2 - (R \leftrightarrow L)\right]$$

$$K_{3sc} = -\frac{\alpha}{\sqrt{2}}\operatorname{Im}\left(A^R_{\perp_1} A^{*R}_{\perp_0} - A^R_{\parallel_1} A^{*R}_{\parallel_0} + (R \leftrightarrow L)\right)$$

$$K_{3s} = -\frac{\alpha}{\sqrt{2}}\operatorname{Im}\left(A^R_{\perp_1} A^{*R}_{\parallel_0} - A^R_{\parallel_1} A^{*R}_{\perp_0} + (R \leftrightarrow L)\right)$$

$$K_{4sc} = \frac{\alpha}{\sqrt{2}}\operatorname{Re}\left(A^R_{\perp_1} A^{*R}_{\perp_0} - A^R_{\parallel_1} A^{*R}_{\parallel_0} + (R \leftrightarrow L)\right)$$

$$K_{4s} = \frac{\alpha}{\sqrt{2}}\operatorname{Re}\left(A^R_{\perp_1} A^{*R}_{\parallel_0} - A^R_{\parallel_1} A^{*R}_{\perp_0} - (R \leftrightarrow L)\right)$$

At Low Recoil: Amplitudes — when $q^2 = O\left(m_b^2\right)$

$$A_{\perp_1}^{L(R)} = -2NC_+^{L(R)} f_\perp^V \sqrt{s_-} \qquad A_{\parallel_1}^{L(R)} = +2NC_-^{L(R)} f_\perp^A \sqrt{s_+}$$

$$A_{\perp_0}^{L(R)} = +\sqrt{2}NC_+^{L(R)} f_0^V \frac{m_{\Lambda_b} + m_\Lambda}{\sqrt{q^2}} \sqrt{s_-} \qquad A_{\parallel_0}^{L(R)} = -\sqrt{2}NC_-^{L(R)} f_0^A \frac{m_{\Lambda_b} - m_\Lambda}{\sqrt{q^2}} \sqrt{s_+}$$

with $s_\pm = (m_{\Lambda_b} \pm m_\Lambda)^2 - q^2$

$$C_+^{R(L)} = \left((\mathcal{C}_9 + \mathcal{C}_{9'}) + \frac{2\kappa m_b m_{\Lambda_b}}{q^2}(\mathcal{C}_7 + \mathcal{C}_{7'}) \pm (\mathcal{C}_{10} + \mathcal{C}_{10'})\right)$$

$$C_-^{R(L)} = \left((\mathcal{C}_9 - \mathcal{C}_{9'}) + \frac{2\kappa m_b m_{\Lambda_b}}{q^2}(\mathcal{C}_7 - \mathcal{C}_{7'}) \pm (\mathcal{C}_{10} - \mathcal{C}_{10'})\right)$$

κ as in [Grinstein/Pirjol hep-ph/0404250]

$$
\begin{aligned}
\rho_1^{\pm} &= |\mathcal{C}_{79} \pm \mathcal{C}_{7'9'}|^2 + |\mathcal{C}_{10} \pm \mathcal{C}_{10'}|^2 \\
\rho_2 &= \text{Re}\left(\mathcal{C}_{79}\mathcal{C}_{10}^* - \mathcal{C}_{7'9'}\mathcal{C}_{10'}^*\right) - i\,\text{Im}\left(\mathcal{C}_{79}\mathcal{C}_{7'9'}^* + \mathcal{C}_{10}\mathcal{C}_{10'}^*\right) \\
\rho_3^{\pm} &= 2\,\text{Re}\left((\mathcal{C}_{79} \pm \mathcal{C}_{7'9'})(\mathcal{C}_{10} \pm \mathcal{C}_{10'})^*\right) \\
\rho_4 &= \left(|\mathcal{C}_{79}|^2 - |\mathcal{C}_{7'9'}|^2 + |\mathcal{C}_{10}|^2 - |\mathcal{C}_{10'}|^2\right) - i\,\text{Im}\left(\mathcal{C}_{79}\mathcal{C}_{10'}^* - \mathcal{C}_{7'9'}\mathcal{C}_{10}^*\right) .
\end{aligned}
$$

Form Factor Uncertainties

- reproduce $\xi_{1,2}$ data points from the lattice [Detmold/Liu/Meinel/Wingate 1212.4827]
  - two points: $q^2 = 13.5\text{GeV}^2$ and $q^2 = 20.5\text{GeV}^2$
  - correlation across q^2 taken into account
  - correlation across $\xi_{1,2}$ not taken into account
- at subleading power, 6 further FFs emerge: $\chi_{1,\ldots,6}$
  - order of magnitude $\sim m_\Lambda / m_{\Lambda_b}$
  - use uncorrelated gaussian priors