



Impact of leptonic τ decays on the distribution of $\bar{B} \rightarrow D\mu\bar{\nu}$ decays

Danny van Dyk
Universität Zürich

based on 1602.06143 with Marzia Bordone and Gino Isidori

Theorieseminar
Dortmund, 31.03.2016



**University of
Zurich** ^{UZH}

Physik Institut

Motivation



Setting the stage

- semileptonic $b \rightarrow c$ transitions long-time focus of particle physics
 - accessible at B factories BaBar and Belle (II), and most recently LHCb
 - allow to constrain CKM element V_{cb}
 - confirmation of KM mechanism lead to nobel prize for KM in 2008
- over recent years, two further points of interest developed
 - tension between extracted value of $|V_{cb}|$ from exclusive and inclusive decays
 - ratio exclusive $b \rightarrow c\tau\nu$ / exclusive $b \rightarrow c\mu\nu$ exceeds SM expectations

five slides to illustrate these points of interest



Inclusive vs. exclusive

Inclusive decays

- requires knowledge of hadronic tensor

$$W^{\mu\nu} \equiv \frac{1}{M_B} \int d^4x e^{-iq \cdot x} \text{Im} \langle \bar{B} | \mathcal{T} \{ \bar{b}(x) \gamma^\mu P_L c(x), \bar{c}(0) \gamma^\nu P_L b(0) \} | \bar{B} \rangle$$

- operator product expansion in α_s , $1/m_b$ and $1/m_c$ yields (schematically)

$$\Gamma(\bar{B} \rightarrow X_c \mu \bar{\nu}) = \frac{G_F^2 m_b^5}{192 \pi^3} |V_{cb}|^2 \left[a_0 \left(1 + \frac{\mu_\pi^2}{2m_b^2} \right) + a_2 \frac{\mu_G^2}{2m_b^2} + a_3 \frac{\bar{\rho}_3}{2m_b^3} + a_4 \frac{\bar{\rho}_4}{2m_b^4} \right]$$

- coefficients a_k are **perturbatively expanded in α_s**

- a_0 known up to $O(\alpha_s^2)$
- a_2 recently calculated up to $O(\alpha_s)$
- $\bar{\rho}_3$ represents two further operators
- $\bar{\rho}_4$ represents seven additional operators

[van Ritbergen 1999; Pak, Czarnecki 2008; Melnikov 2008]

[Mannel, Pivovarov, Rosenthal 1506.08167]

[Mannel, Turczyk, Uraltsev 1009.4622]

- good rate of convergence of and within coefficients a_k
 - good theory control in extraction of $|V_{cb}|$



Inclusive vs. exclusive

Exclusive decays

- exclusive semileptonic decays follow entirely different approach
- need hadronic matrix element (HME)

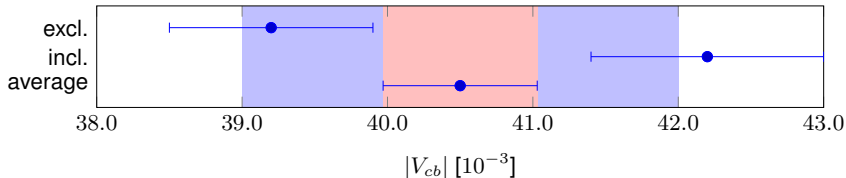
$$q^2 \equiv (p - k)^2$$

$$\langle D(k) | \bar{c} \gamma^\mu b | \bar{B}(p) \rangle = f_+(q^2) \left[(p + k)^\mu - \frac{M_B^2 - M_D^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M_B^2 - M_D^2}{q^2} q^\mu$$

- similarly but more complicated for D^* , all four D^{**} states, etc.
- V_{cb} only extracted from D and D^* , since their HMEs most reliable
- form factors f_+ , f_0 only accessible through **non-perturbative** methods
 - lattice QCD [e.g. Heechang, Bouchard, Lepage, Monahan, Shigemitsu 1505.03925]
 - QCD sum rules on the light-cone with B-meson LCDAs [Faller, Khodjamirian, Klein, Mannel 0809.0222]
 - QCD sum rules at zero hadronic recoil [Bigi, Shifman, Uraltsev, Vainshtein hep-ph/9405410]



Inclusive vs. exclusive



p value: 0.33%

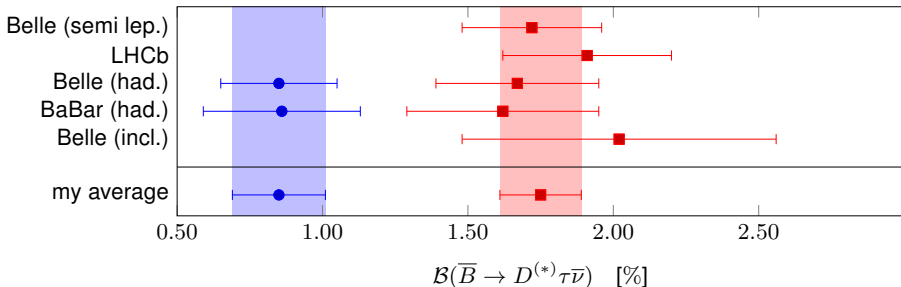
naive weighted average, average w/ scale factor

- extracted values exhibit 99.67% $\approx 3\sigma$ tension [PDG 2014 & 2015 part. upd.]
- in addition: sum of exclusive semileptonic B s does not saturate inclusive measurement
 - $\overline{B} \rightarrow D^{(*)} \mu \overline{\nu}$ might close the gap [Bernlochner,Ligeti,Turczyk 1202.1834]



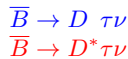
Exclusive semitauonic decays

data since 2007



sum of excl. modes: $(2.60 \pm 0.21) \%$

some entries calculated using world averages on $\mathcal{B}(\bar{B} \rightarrow D^{(*)} \mu \bar{\nu})$

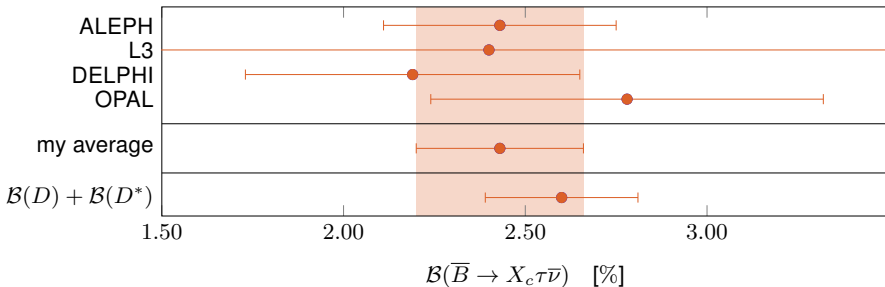


[Belle (incl.) 0706.4429; BaBar (had.) 0709.1698; Belle (had.) 1507.03233; LHCb 1506.08614; Belle (semi lep.) 1603.06711]



Inclusive semitauonic decays

from LEP experiments



– sum of $D^{(*)}$ modes saturates inclusive rate of $(2.43 \pm 0.23)\%$

– very little space for orbitally excited D^{**} modes,

– $B(\bar{B} \rightarrow D^{**} \mu \nu) \approx 0.79\%$

assuming $B(D^{**} \rightarrow D^{(*)} \pi) = 1$

– maybe $R_{D^{**}}$ suppressed?

– updated inclusive measurement needed to settle questions (\rightarrow Belle II)

[OPAL hep-ex/0108031; DELPHI Phys.Lett. B496 (200) 43-48; L3 Z. Phys. C71 (1996) 379-390; ALEPH hep-ex/0010022]



Reasons?

(personally biased) list of possible reasons

$V_{cb}, R_{D^{(*)}}$ **lattice QCD** does not understand the form factors well enough

- underestimating form factors for time-like polarization?

V_{cb} model dependence of **experimental** q^2 spectrum in $\bar{B} \rightarrow D\mu\bar{\nu}$

- existing results explicitly use HQET prediction in fitting the q^2 spectrum
- new Belle (semi lep.) result is the only exception more info in backups

$V_{cb}, R_{D^{(*)}}$ $\bar{B} \rightarrow D^{(*)}\tau(\rightarrow \mu\bar{\nu}\nu)\bar{\nu}$ background not well-enough understood

- **focus of the following exploratory study**
- using $\bar{B} \rightarrow D\tau\bar{\nu}$ due to its simple hadronic matrix element



University of
Zurich^{UZH}

Physik Institut

Impact of leptonic τ decays



Experimental PoV

- no **hard** criterium to distinguish $\bar{B} \rightarrow D\mu\bar{\nu}$ from $\bar{B} \rightarrow D\tau(\rightarrow \mu\bar{\nu}\nu)\bar{\nu}$
- experiment sees **neutrino-inclusive** decay rate

$$\begin{aligned} \frac{d\Gamma(\bar{B} \rightarrow D\mu X_{\bar{\nu}})}{dq^2 d\cos\theta_{[\mu]}} &\equiv \frac{d\Gamma(\bar{B} \rightarrow D\mu\bar{\nu}_{\mu})}{dq^2 d\cos\theta_{[\mu]}} + \frac{d\Gamma(\bar{B} \rightarrow D\tau(\rightarrow \mu\bar{\nu}_{\mu}\nu_{\tau})\bar{\nu}_{\tau})}{dq^2 d\cos\theta_{[\mu]}} \\ &\equiv \frac{d\Gamma_1}{dq^2 d\cos\theta_{[\mu]}} + \frac{d\Gamma_3}{dq^2 d\cos\theta_{[\mu]}} \end{aligned}$$

- analytical results for Γ_3 not used in experimental analysis
- first considered in this work
 - for NP contributions see also [\[Alonso,Kobach,Camalich 1602.07671\]](#)
- varying means to statistically disentangle both decays
 - Belle (II) uses NeuroBayes (\rightarrow black box)



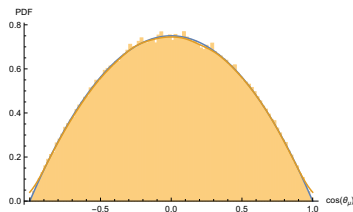
Signal 1: $\overline{B} \rightarrow D\mu\overline{\nu}$

$$\overline{B}(p) \rightarrow D(k) \mu(q_{[\mu]}) \overline{\nu}(q_{[\overline{\nu}\mu]})$$

- pseudo-scalar to pseudo-scalar transition
 - one hadronic form factors f_+
 - second form factor f_0 enters only m_μ^2/q^2 suppressed
- simple kinematics:
 - momentum transfer $q^2 = (p - k)^2$
 - muon helicity angle $\cos\theta_{[\mu]}$ in $\mu\overline{\nu}$ RF
- distribution in $\theta_{[\mu]}$

$$\begin{aligned} \frac{d^2\Gamma}{dq^2 d\cos\theta_{[\mu]}} &= a + b \cos\theta_{[\mu]} + c \cos^2\theta_{[\mu]} \\ &= a \sin^2\theta_{[\mu]} + O(m_\mu^2/q^2) \end{aligned}$$

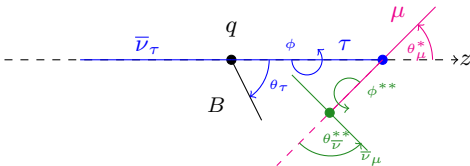
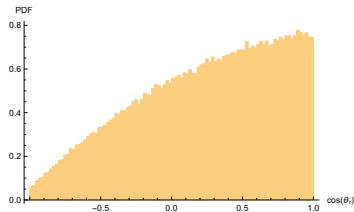
- a, b, c : functions of q^2



Signal 2: $\bar{B} \rightarrow D\tau(\rightarrow \mu\bar{\nu}\nu)\bar{\nu}$

$$\bar{B}(p) \rightarrow D(k) \mu(q_{[\mu]}) \bar{\nu}_\mu(q_{[\bar{\nu}_\mu]}) \nu_\tau(q_{[\nu_\tau]}) \bar{\nu}_\tau(q_{[\bar{\nu}_\tau]})$$

- hadronic matrix element
 - both form factors contribute
- complicated kinematics:
 - momentum transfer $q^2 = (p - k)^2$
 - $\bar{\nu}\nu$ mass $q_{[\nu_\tau\bar{\nu}_\mu]}^2$
 - five angles

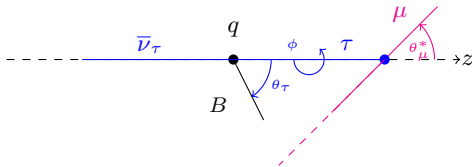
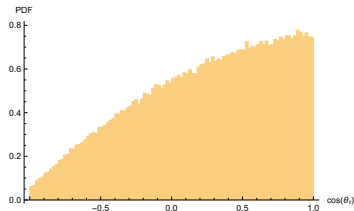




Signal 2: $\bar{B} \rightarrow D\tau(\rightarrow \mu\bar{\nu}\nu)\bar{\nu}$

$$\bar{B}(p) \rightarrow D(k) \mu(q_{[\mu]}) \bar{\nu}_\mu(q_{[\bar{\nu}_\mu]}) \nu_\tau(q_{[\nu_\tau]}) \bar{\nu}_\tau(q_{[\bar{\nu}_\tau]})$$

- hadronic matrix element
 - both form factors contribute
- complicated kinematics:
 - momentum transfer $q^2 = (p - k)^2$
 - $\bar{\nu}\nu$ mass $q_{[\nu_\tau\bar{\nu}_\mu]}^2$
 - five angles, **only 3 of which are needed for pheno studies**

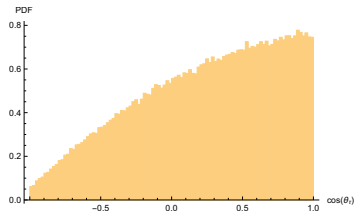




Signal 2: $\bar{B} \rightarrow D\tau(\rightarrow \mu\bar{\nu}\nu)\bar{\nu}$

$$\bar{B}(p) \rightarrow D(k) \mu(q_{[\mu]}) \bar{\nu}_\mu(q_{[\bar{\nu}_\mu]}) \nu_\tau(q_{[\nu_\tau]}) \bar{\nu}_\tau(q_{[\bar{\nu}_\tau]})$$

- hadronic matrix element
 - both form factors contribute
- complicated kinematics:
 - momentum transfer $q^2 = (p - k)^2$
 - $\bar{\nu}\nu$ mass $q_{[\nu_\tau\bar{\nu}_\mu]}^2$
 - five angles, **only 3 of which are needed for pheno studies**



$$\frac{d^5\Gamma_3}{dq^2 dq_{[\nu_\tau\bar{\nu}_\mu]}^2 d^2\Omega d\Omega^*} = \frac{\tilde{\Gamma}_3}{\pi m_\tau^8 q^6} \left[A + B \cos \theta_{[\tau]} + C \cos^2 \theta_{[\tau]} \right. \\ \left. + (D \sin \theta_{[\tau]} + E \sin \theta_{[\tau]} \cos \theta_{[\tau]}) \cos \phi \right]$$

A, \dots, E are functions of q^2 , $q_{[\nu_\tau\bar{\nu}_\mu]}^2$ and $\cos \theta_{[\mu]}^*$

Observables

$\cos \theta_{[\mu]}$: muon helicity angle

- **physical** observable **only** in the 1ν final state
- defined in q rest frame
 - in terms of Lorentz invariants

$$\cos \theta_{[\mu]} \equiv 2 \frac{(q - 2q_{[\mu]}) \cdot k}{\sqrt{\lambda}}$$

- 3ν case: $\theta_{[\mu]} \neq \theta_{[\mu]}^*$
- boundaries

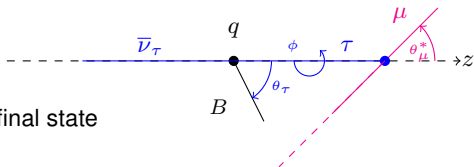
$$-1 \leq \cos \theta_{[\mu]} \leq 1$$

for 1ν final state

$$-1 \leq \cos \theta_{[\mu]} \lesssim 56.7$$

for 3ν final state

- upper bounds very different from each other
- not suitable for **common** parametrization

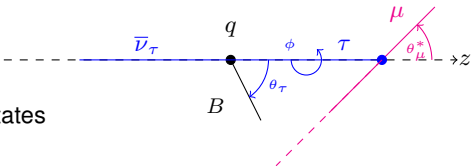


see backups for functional dep.

Observables

E_μ : muon energy in B rest frame

- **physical** observable for both final states
- defined in \overline{B} rest frame
 - in terms of Lorentz invariants



$$E_\mu \equiv \frac{p \cdot q_{[\mu]}}{M_B}$$

- boundaries

$$m_\mu \approx 0 \leq E_\mu \lesssim 2.31 \text{ GeV}$$

for 1ν final state

$$m_\mu \approx 0 \leq E_\mu \lesssim 2.26 \text{ GeV}$$

for 3ν final state

- upper bounds very similar for both final states
- suggests **common** parametrization of E_μ dependence



Neutrino-inclusive decay

aim: obtain normalized, differential decay widths for the neutrino-inclusive decay

- decay widths in terms of $\cos \theta_{[\mu]}$ and E_μ yield complicated expressions

[see Alonso,Kobach,Camalich 1602.07671]

- our approach: Monte Carlo simulations of pseudo events
 - implement signal PDFs in EOS, including dependence on form factor parameters

[DvD et al. <http://github.com/eos/eos>]

- draw $\approx 4 \cdot 10^6$ pseudo events of $(q^2, \cos \theta_{[\mu]})$ for the 1ν final state
- draw $\approx 4 \cdot 10^6$ pseudo events of $(q^2, q_{[\nu\tau\bar{\nu}\mu]}^2, \Omega, \Omega^*)$ for the 3ν final state

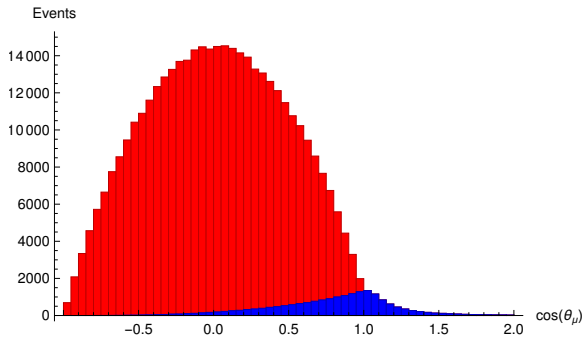
Ω, Ω^* : solid angles

- compute observable of interest for each set of pseudo events
- combine sets with weights

$$\omega_1 = \frac{1}{1 + R_D \mathcal{B}(\tau \rightarrow \mu\nu\bar{\nu})} \quad \omega_3 = 1 - \omega_1$$



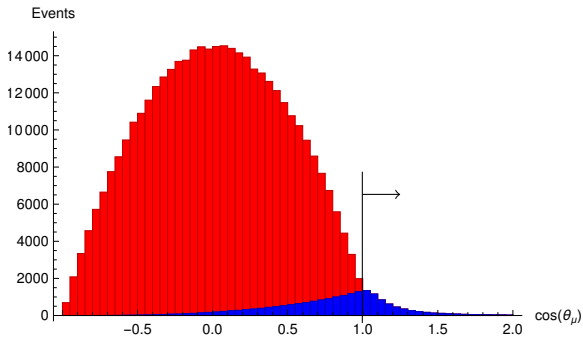
Distribution in $\cos \theta_{[\mu]}$



neutrino-inclusive, 3ν final state



Distribution in $\cos \theta_{[\mu]}$



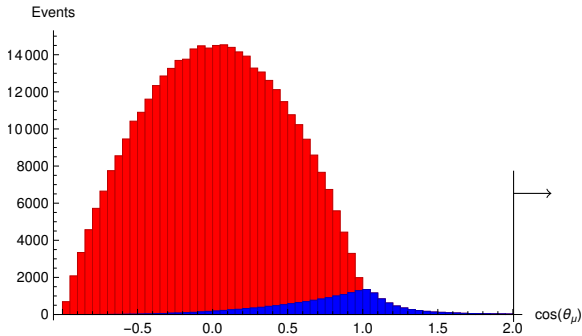
Heavy tails

$\cos \theta_{[\mu]} > 1$ 32.3% of 3ν ev.

neutrino-inclusive, 3ν final state



Distribution in $\cos \theta_{[\mu]}$



Heavy tails

$\cos \theta_{[\mu]} > 1$ 32.3% of 3ν ev.

$\cos \theta_{[\mu]} > 2$ $\approx 2\%$ of 3ν ev.



neutrino-inclusive, 3ν final state



New method to extract $R_D \mathcal{B}(\tau \rightarrow \mu \bar{\nu} \nu)$

- cash in on heavy tail of $B \rightarrow D \mu X_{\bar{\nu}}$, and turn it into new method to extract R_D
- we suggest measurement of

$$\rho_D^{\text{exp}} \equiv \frac{\text{\# of } X_{\nu} \text{ events with } \cos \theta_{\mu} > 1}{\text{total \# of } X_{\nu} \text{ events}}$$

- precise calculation possible for

$$\rho_D^0 \equiv \frac{\text{\# of } 3\nu \text{ events with } \cos \theta_{\mu} > 1}{\text{total \# of } 3\nu \text{ events}} = 0.323 \pm 0.002 \quad (0.6\%)$$

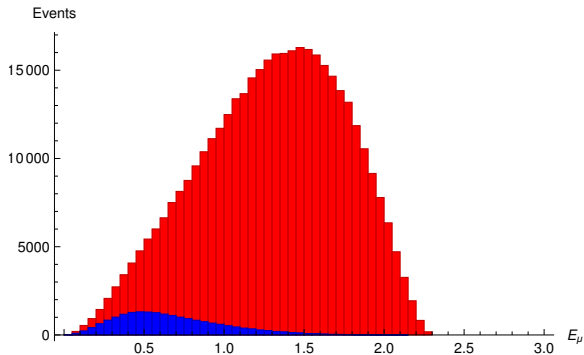
- uncertainty statistical
- parametric uncertainty (form factors) within ± 0.002
- combine to extract

$$R_D \mathcal{B}(\tau \rightarrow \mu \bar{\nu} \nu) = \frac{\rho_D^{\text{exp}}}{\rho_D^0 - \rho_D^{\text{exp}}}$$

- will probably also work for other charmed hadrons: D^* , Λ_c



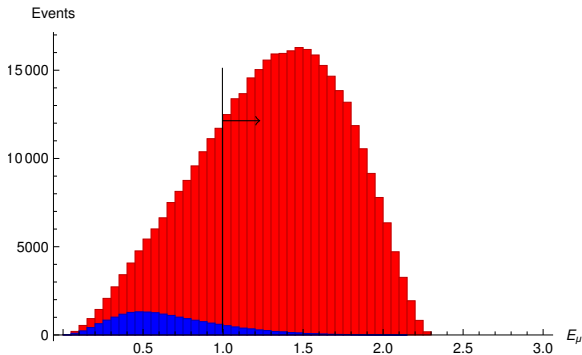
Distribution in E_μ



neutrino-inclusive, 3ν final state



Distribution in E_μ



neutrino-inclusive, 3ν final state

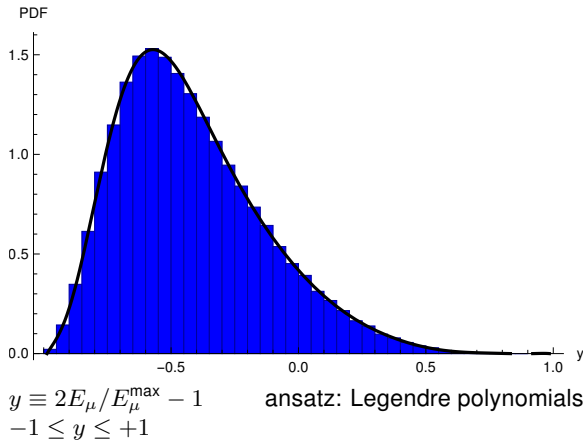
Background

$E_\mu > 1$ $\approx 15\%$ of 3ν ev.
 $\approx 0.8\%$ background

- parametrization of background PDF (and numeric results) in the paper
- allows fit of 3ν to 1ν ratio to data



Distribution in E_μ



Background

$$E_\mu > 1 \quad \approx 15\% \text{ of } 3\nu \text{ ev.} \\ \approx 0.8\% \text{ background}$$

- parametrization of background PDF (and numeric results) in the paper
- allows fit of 3ν to 1ν ratio to data



Implications for extraction of $|V_{cb}|$

– for last slide, let's assume $R_D \neq R_D^{\text{SM}}$, and let $\Delta R_D = R_D - R_D^{\text{SM}}$

– influences $|V_{cb}|^{\text{incl}}$ extraction

– in simulation of $B \rightarrow X_c \mu \bar{\nu}$ backgrounds $R_{D^{(*)}}^{\text{SM}}$ is used

– let $\Delta|V_{cb}|^{(\text{incl.})} = |V_{cb}|^{(\text{exp})} - |V_{cb}|^{(\text{true})}$

– we find for the maximal shift

$$\frac{\Delta|V_{cb}|^{(\text{incl.})}}{|V_{cb}|} = \frac{1}{2} \Delta R_D \mathcal{B}(\tau \rightarrow \mu \bar{\nu} \nu) \approx 0.9\%$$

– not far from combined theory and experimental error on $|V_{cb}|^{(\text{incl.})}$

[PDG 2014 and 2015 part. upd.]

– important to study in more detail for future Belle II analyses



Summary and outlook

- lots of interesting things happening in b -physics right now
 $b \rightarrow c\tau\nu$ definitely among those
- we investigated $\overline{B} \rightarrow D\tau(\rightarrow \mu\bar{\nu}\nu)\bar{\nu}$ and its impact on the extraction of V_{cb} and R_D
 - analytic results can be used to check experimental Monte Carlo studies
 - numeric results for PDF of the 3ν background
- outlined a new method to extract R_D from $\cos\theta_{[\mu]}$ distribution of the neutrino-inclusive decay
- currently adapting method to $\overline{B} \rightarrow D^*\tau\nu$, with emphasis on underconstrained \overline{B} momentum at LHCb

[Bordone,Chraszcz,DvD w.i.p.]



**University of
Zurich** ^{UZH}

Physik Institut

Appendix



Model-dependence of exclusive $\bar{B} \rightarrow D\mu\bar{\nu}$ measurements

- q^2 spectrum of the decay is relevant to extraction of V_{cb} and form factor ratios (\rightarrow crosscheck of R_D inputs)
- Belle analyses **do not provide** histograms of observables as functions of q^2
[e.g. Belle hep-ex/0111082]
 - (personally) could not find BaBar analysis that do, either!
 - only fits of HQET-inspired parametrization (to **leading power** in $1/m_b!$) are available
 - ditto for $\bar{B} \rightarrow D^*$
- according to sources within Belle, reanalysis of the data is not possible
 - new Belle analyses do provide the “raw” q^2 spectrum [e.g. Belle (semi lep.) 1603.06711]



Observables

$$1\nu \quad E_\mu \Big|_{1\nu} = \frac{1}{4M_B} \left[(M_B^2 - M_P^2 + q^2) - \sqrt{\lambda} \cos \theta_\mu \right],$$

$$3\nu \quad \cos \theta_{[\mu]} \Big|_{3\nu} = 2\beta_{\nu\bar{\nu}} \left\{ \left(\frac{(1 - 2\beta_{\nu\bar{\nu}})}{\beta_{\nu\bar{\nu}}} + 2\beta_\tau \right) \frac{M_B^2 - M_P^2 - q^2}{2\sqrt{\lambda}} + \beta_\tau \cos \theta_{[\tau]} \right. \\ \left. - \left(2\beta_\tau \frac{M_B^2 - M_P^2 - q^2}{2\sqrt{\lambda}} - (1 - \beta_\tau) \cos \theta_{[\tau]} \right) \cos \theta_{[\mu]}^* - \sqrt{1 - 2\beta_\tau} \sin \theta_{[\mu]}^* \sin \theta_{[\tau]} \cos \phi \right\}$$

$$E_\mu \Big|_{3\nu} = \frac{\beta_{\nu\bar{\nu}}}{2M_B} \left[(M_B^2 - M_P^2 + q^2) ((1 - \beta_\tau) + \beta_\tau \cos \theta_{[\mu]}^*) \right. \\ \left. - \sqrt{\lambda} (\beta_\tau + (1 - \beta_\tau) \cos \theta_{[\mu]}^*) \cos \theta_{[\tau]} + \sqrt{1 - 2\beta_\tau} \sqrt{\lambda} \sin \theta_{[\mu]}^* \sin \theta_{[\tau]} \cos \phi \right]$$

with $\lambda = \lambda(M_B^2, M_D^2, q^2)$ the Källèn function



The case of $\bar{B} \rightarrow \pi\tau\bar{\nu}$

- $D \rightarrow \pi$ easy enough
- however, small mass of π makes for some numerical changes
- tail $\cos\theta_{[\mu]} > 1$ very light: $\approx 3.3\%$
new method will probably not work for pions
- distribution in E_μ broader
 - $R_\pi \approx 0.7$ larger, thus control of subtraction much more important!
 - background PDF parametrization should work as well as for D

