K-Matrix for Charmonium Spectroscopy

MIAPP

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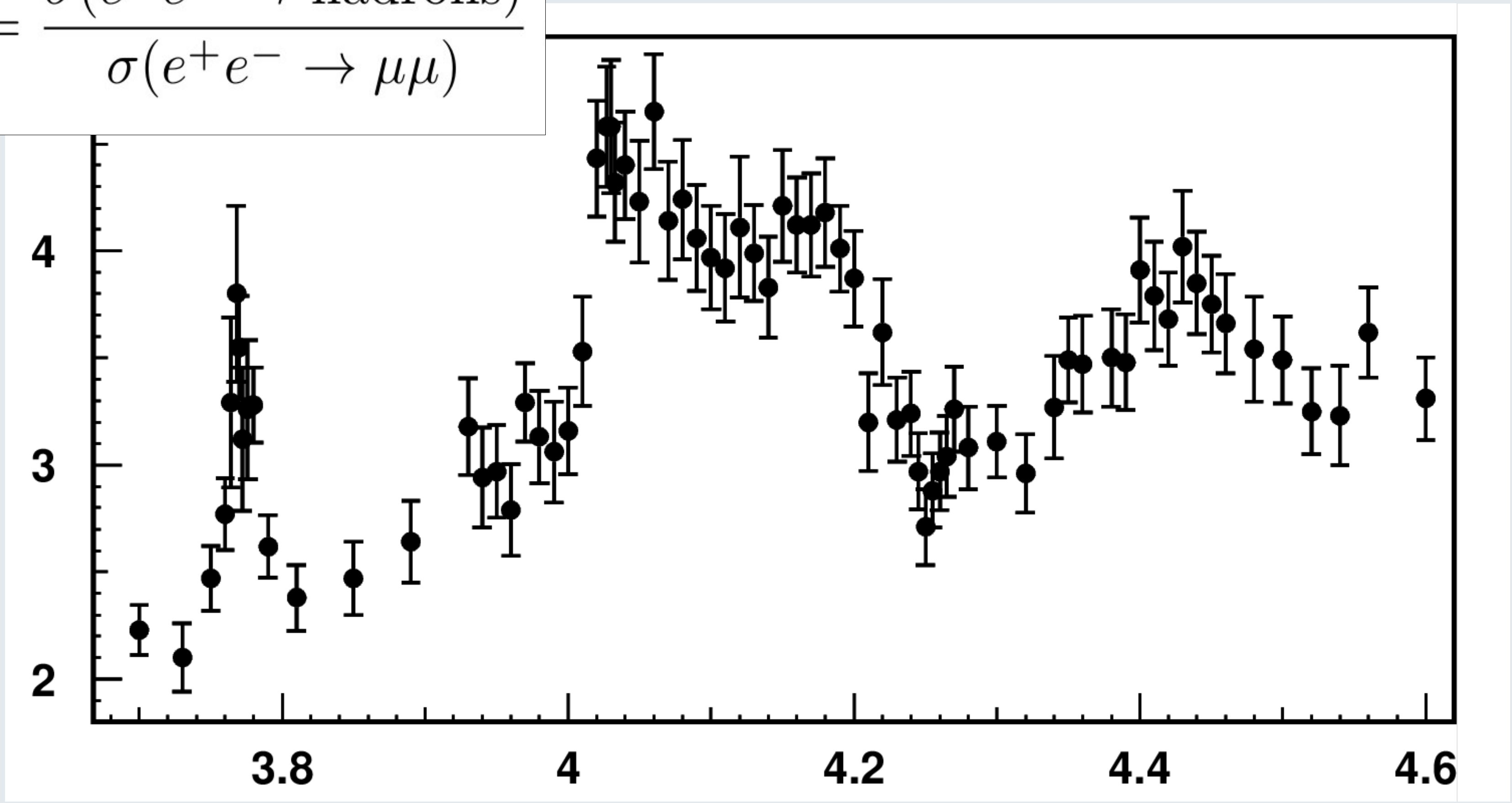
The R ratio

[BES '01]

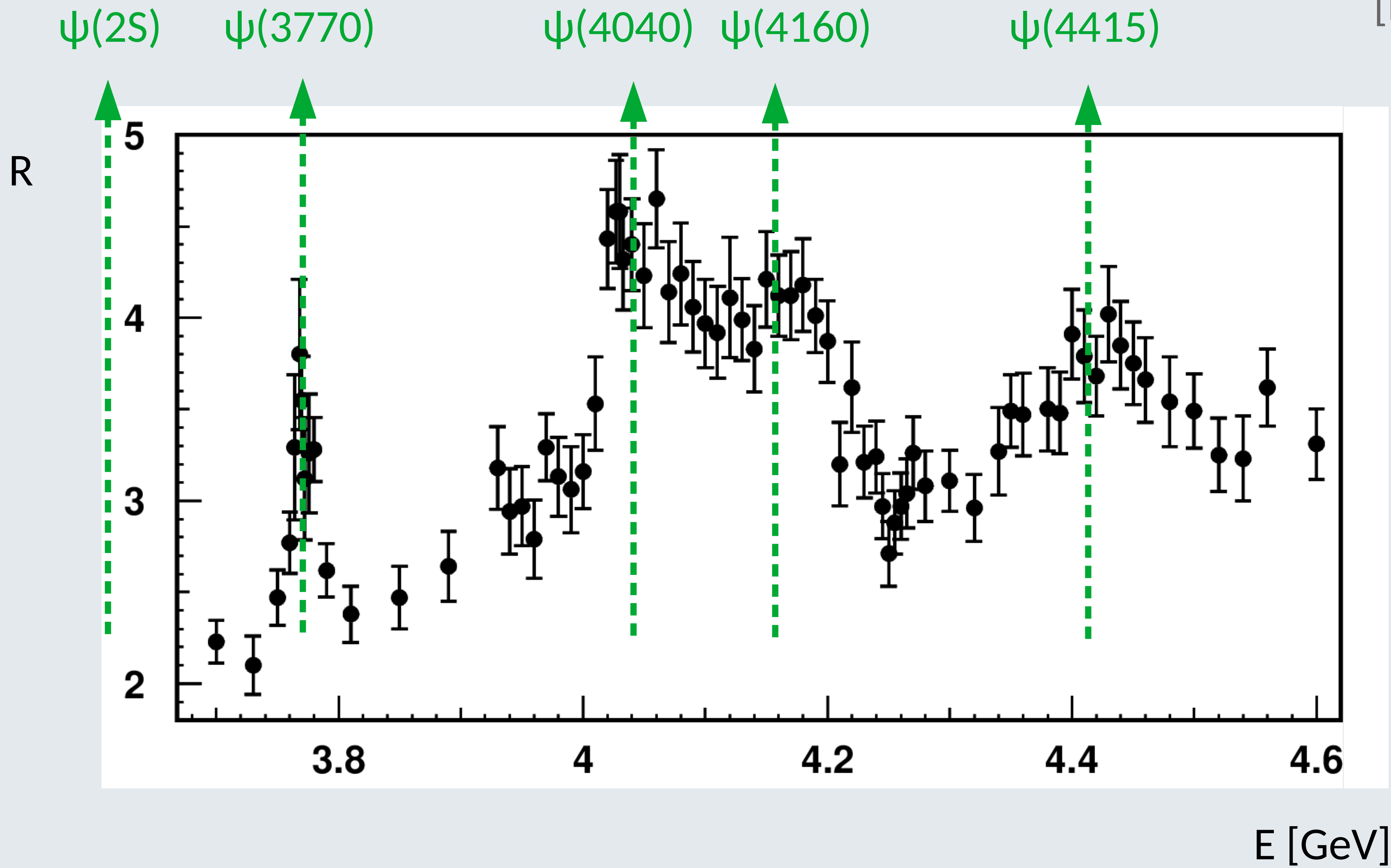
$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu\mu)}$$



E [GeV]

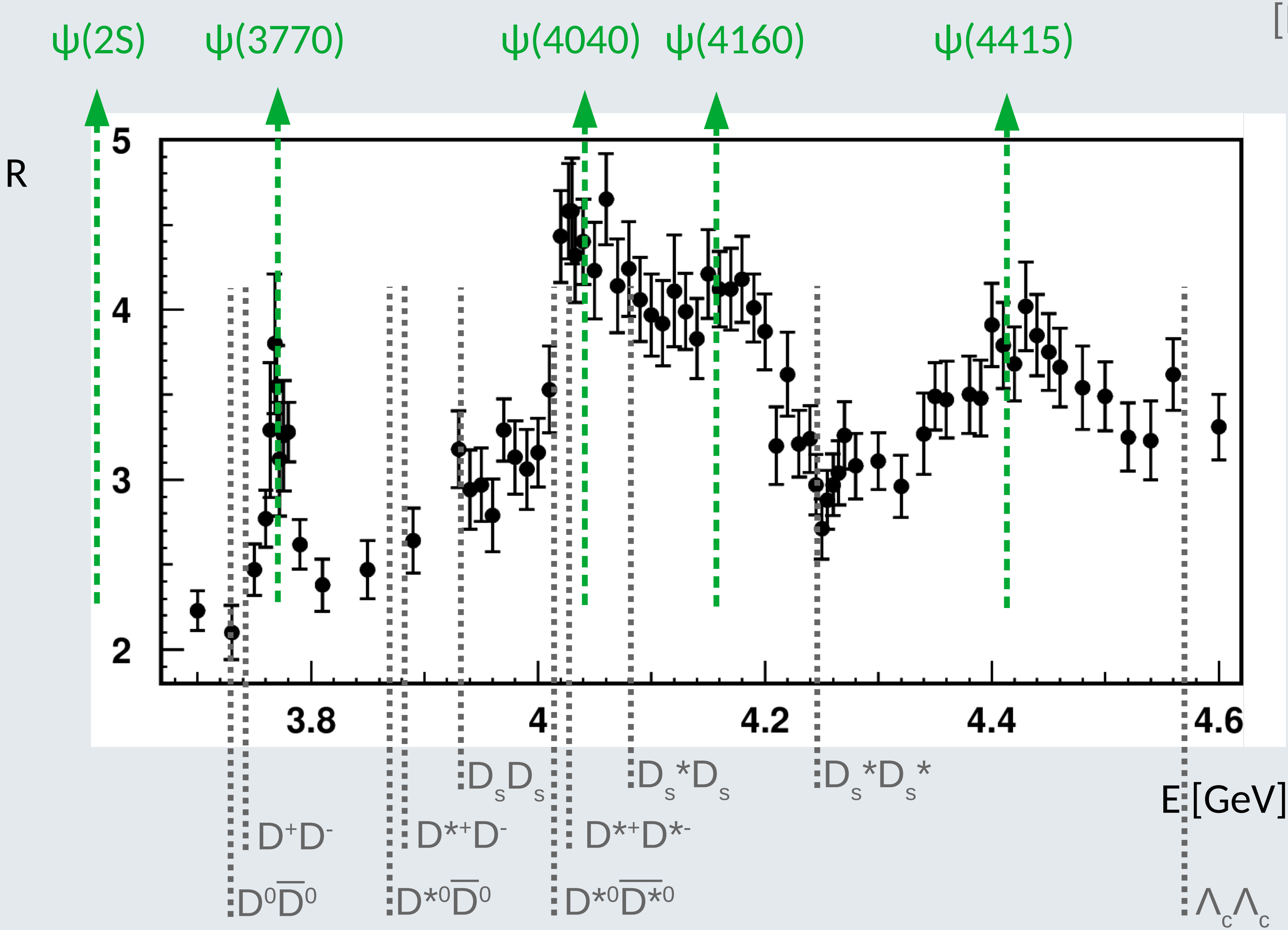
The main $I^G(J^{PC}) = 0^-(1^{--})$ resonances

[BES '01]



Thresholds

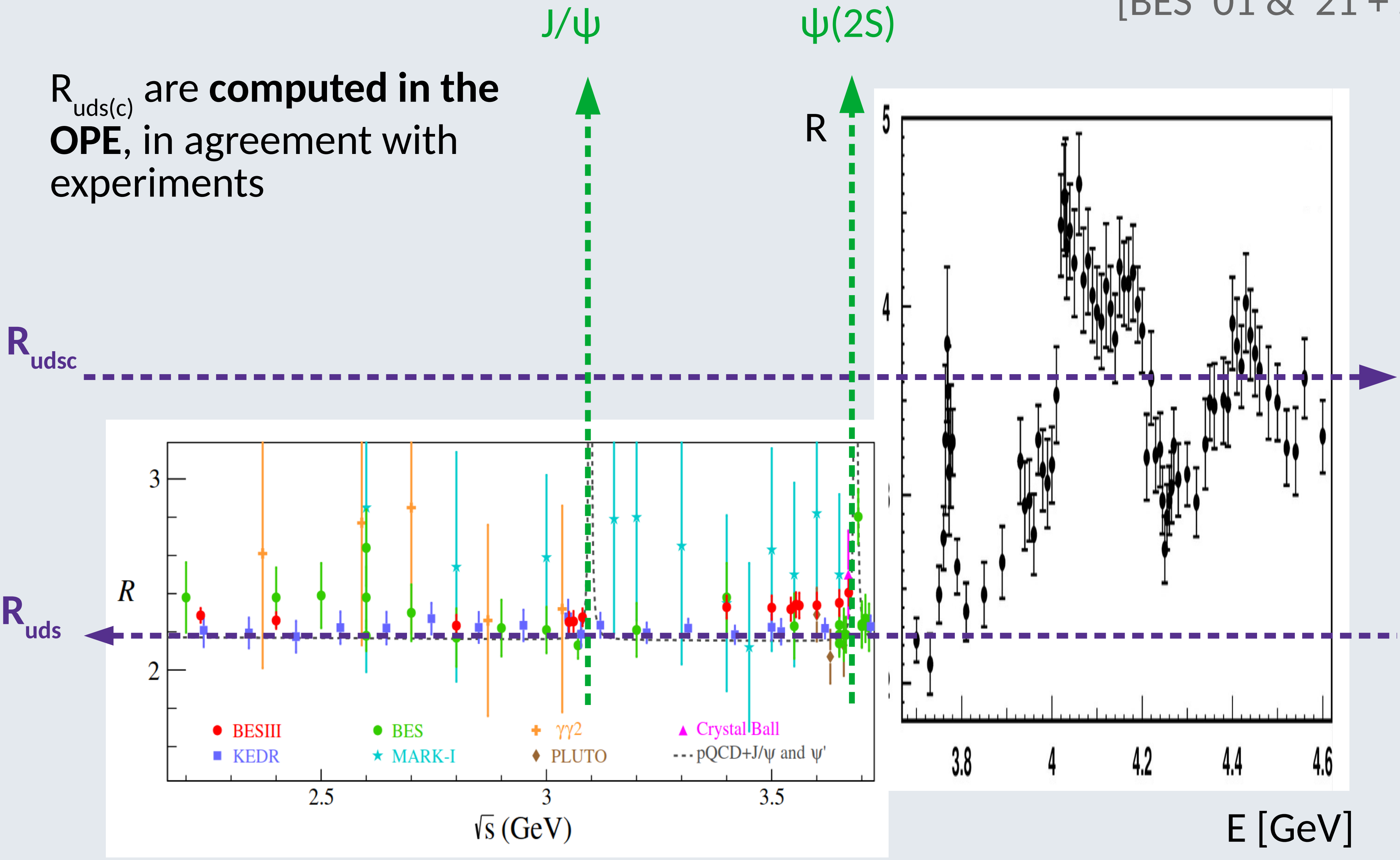
[BES '01]



Limits

[BES '01 & '21 + ...]

$R_{uds(c)}$ are **computed in the OPE**, in agreement with experiments



Experimental fit to the R ratio only

Breit-Wigner ansatz:

[BES '07, Lyon & Zwicky '14, Braß et al. '17]

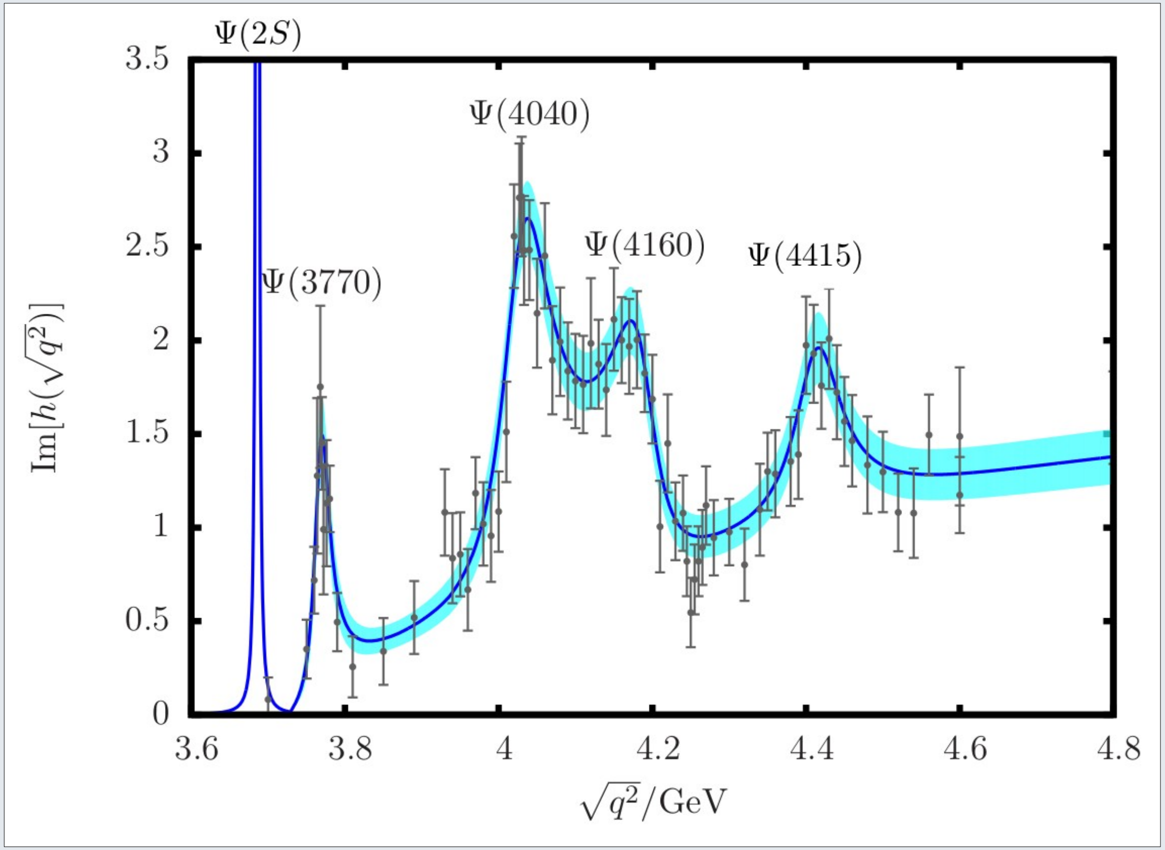
$$\mathcal{T}_r^f(W) = \frac{M_r\sqrt{\Gamma_r^{ee}\Gamma_r^f}}{W^2 - M_r^2 + iM_r\Gamma_r} e^{i\delta_r}$$

$$\Gamma_r^f(W) = \hat{\Gamma}_r \frac{2M_r}{M_r + W} \sum_L \frac{Z_f^{2L+1}}{B_L},$$

- Masses (4)
- Total widths (4)
- Electron widths (4)
- Phases (3)
- Background (1)

$\rightarrow$ **16 parameters**

$\rightarrow$ **p-value: 44%**



What data do we have?

Two main experimental approaches:

- **Fixed energy scans** (CLEO, BES…)
  - Inclusive measurement

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu\mu)}$$

  - Exclusive cross-sections
    e.g. $e^+e^- \to D^{(*)}\overline{D}^{(*)}$, $D_s^{(*)}\overline{D}_s^{(*)}$, $\Lambda_c\Lambda_c$

- **ISR analysis** (BaBar, Belle)
  - mostly exclusive cross-sections
    e.g. $e^+e^- \to D^{(*)}\overline{D}^{(*)}$, $D_s^{(*)}\overline{D}_s^{(*)}$, $\Lambda_c\Lambda_c$
  - one helicity analysis
    $e^+e^- \to D_L^*\overline{D}_L^*$, $D_L^*\overline{D}_T^*$, $D_T^*\overline{D}_T^*$

Experimental fit to the R ratio only

→ The Breit-Wigner approach **does not generalize well:**

- Every channel adds 16 **new parameters**
- **Unitarity** can be violated
- Partial widths are **disconnected** from the R ratio width

K-matrix approach

- We have a **coupled multichannel problem:**

  $\psi \to e^+e^-$, $\psi \to D^{(*)}\overline{D}^{(*)}$, $(\psi \to BK^{(*)})$

- Resonances are **close to thresholds**
- **K-matrix** is the tool to use [Chung, Brose *et al.* '95]

$$S = 1 + 2i\,T = 1 + 2i\,\rho^{1/2}\,\hat{T}\,\rho^{1/2}$$

$$\hat{T} = \hat{K}\,(1 - i\rho\,\hat{K})^{-1}$$

$$\hat{K}_{ij} = \sum_{\psi_r} \frac{g^0_{ri}\,g^0_{rj}}{m^2_{\psi_r} - q^2} + \hat{c}_{ij}$$

Real valued couplings

Phase space factors

ee and DD channels

$c\bar{c}$ resonances

Non-resonant contributions

K-matrix approach

- We have a **coupled multichannel problem:**

  $\psi \to e^+e^-$, $\psi \to D^{(*)}\overline{D}^{(*)}$, $(\psi \to BK^{(*)})$

- Resonances are **close to thresholds**
- **K-matrix** is the tool to use [Chung, Brose *et al.* '95]

- **Problem**: other decays contribute, including 3-body decays

  e.g. $\psi(3770) \to J/\psi\, \pi\, \pi$

  → How to treat it?

  Possible solution: approximate the width due to these decays through **uncoupled effective** 2-body channels (one per resonance)

List of channels

Dilepton channel (assumes LFU)

	channel	type	related to channel
0	e^+e^-	PP (P wave)	-
1	eff(2S)	Effective	-
2	eff(3770)	Effective	-
3	eff(4040)	Effective	-
4	eff(4160)	Effective	-
5	eff(4415)	Effective	-
6	$D^0 \; \overline{D^0}$	PP (P wave)	-
7	$D^+ \; D^-$	PP (P wave)	6 (isospin)
8	$D^0 \; \overline{D^{*0}}$	VP (P wave)	-
9	$D^{*0} \; \overline{D^0}$	VP (P wave)	8 (c.c.)
10	$D^+ \; D^{*-}$	VP (P wave)	8 (isospin)
11	$D^{*+} \; D^-$	VP (P wave)	8 (c.c.)
12	$D_s^+ \; D_s^-$	PP (P wave)	- (*)
13	$D^{*0} \; \overline{D^{*0}}$	VV (P wave, S=0)	-
14	$D^{*0} \; \overline{D^{*0}}$	VV (P wave, S=2)	-
15	$D^{*0} \; \overline{D^{*0}}$	VV (F wave, S=2)	-
16	$D^{*+} \; D^{*-}$	VV (P wave, S=0)	13 (isospin)
17	$D^{*+} \; D^{*-}$	VV (P wave, S=2)	14 (isospin)
18	$D^{*+} \; D^{*-}$	VV (F wave, S=2)	15 (isospin)
19	$D_s^+ \; D_s^{*-}$	VP (P wave)	- (*)
20	$D_s^{*+} \; D_s^-$	VP (P wave)	19 (c.c.)
21	$D_s^{*+} \; D_s^{*-}$	VV (P wave, S=0)	- (*)
22	$D_s^{*+} \; D_s^{*-}$	VV (P wave, S=2)	- (*)
23	$D_s^{*+} \; D_s^{*-}$	VV (F wave, S=2)	- (*)

Effective channels (fix the resonances widths)

$D_{(s)}\overline{D}_{(s)}$ channels

$D_{(s)}\overline{D}^*_{(s)}$ channels

$D^*_{(s)}\overline{D}^*_{(s)}$ channels

(*) SU(3) symmetry could be imposed

Centrifugal barrier factors (finite size effects)

[Blatt & Weisskopf '52]

$$\hat{K}_{ij} = \sum_{\psi_r} \frac{g^0_{ri}\, g^0_{rj}\, B^L_{ri}(q, q_\alpha) B^L_{rj}(q, q_\alpha)}{m^2_{\psi_r} - q^2} + \hat{c}_{ij}$$

$$B^l_{\alpha i}(q, q_\alpha) = \frac{F_l(q)}{F_l(q_\alpha)}$$

$$F_0(q) = 1$$

$$F_1(q) = \sqrt{\frac{2z}{z+1}}$$

$$F_2(q) = \sqrt{\frac{13z^2}{(z-3)^2 + 9z}}$$

$z = (q/q_R)^2$ and q_R corresponds to the range of interaction.

Preliminary result – General conclusions

- Need for **non-resonant contributions**
  - Allows to account for $R_{udsc} - R_{uds}$; impacts the exclusive channels
  - $\hat{c}_{0j}$ (i.e. involving e^+e^- channel) seem enough to describe the data

$$\hat{K}_{ij} = \sum_{\psi_r} \frac{g^0_{ri}\, g^0_{rj}}{m^2_{\psi_r} - q^2} + \hat{c}_{ij}$$

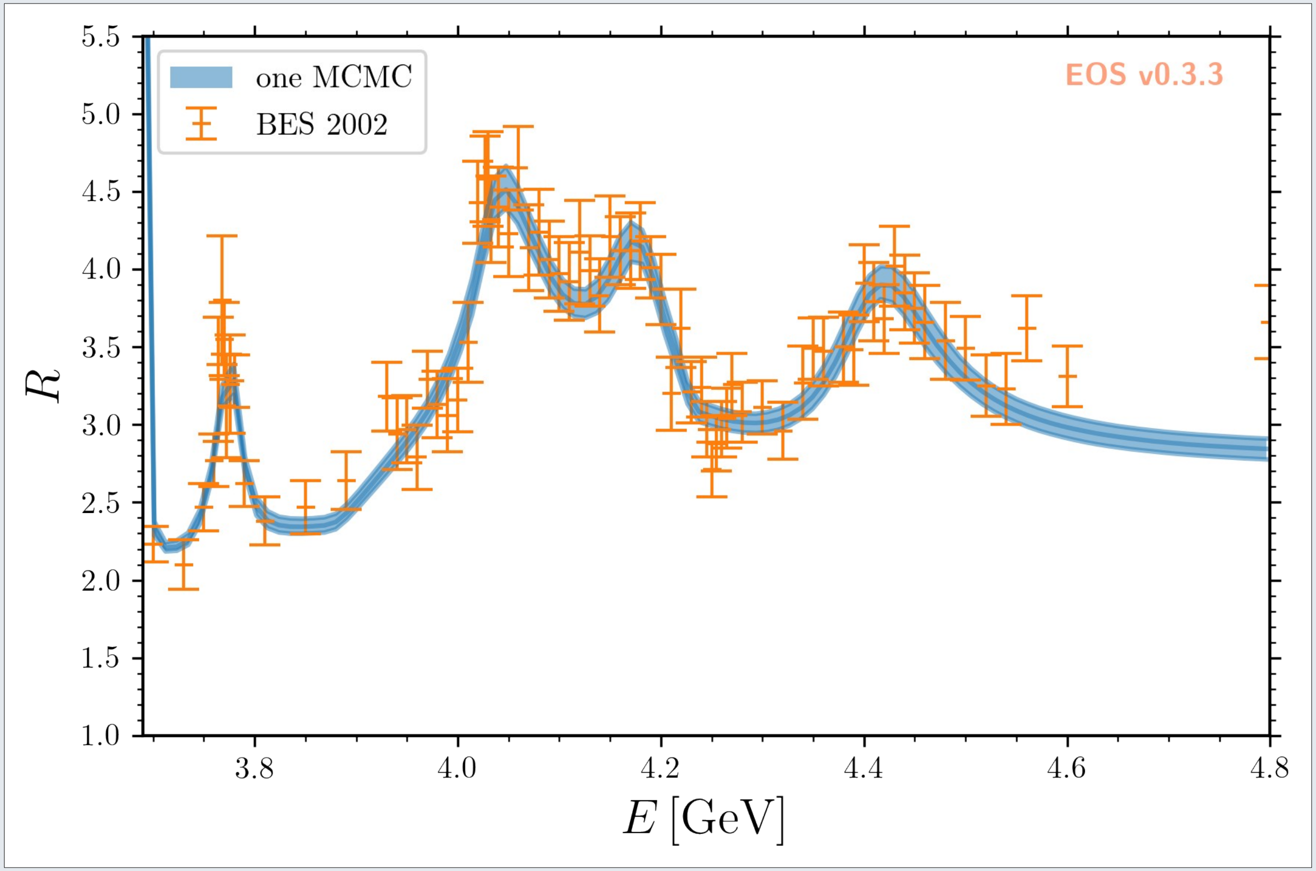
- **Sub-threshold couplings** play a crucial role [Uglov, Kalashnikova *et al.* '19]

- Data seems to be **insufficient to determine all parameters** → needs of assumptions:
  - Isospin relates $D^0\overline{D}^0$ to D^+D^-
  - SU(3) would relate $D^0\overline{D}^0$ to $D_s^+D_s^-$
  - PDG couplings to e^+e^- **(from lattice in the future?)**

Preliminary result – Numerics

The fit converges!

- **76 parameters, p-value ~ 10%**
- Uncertainties are estimated using **Monte-Carlo techniques**



Preliminary result – Numerics

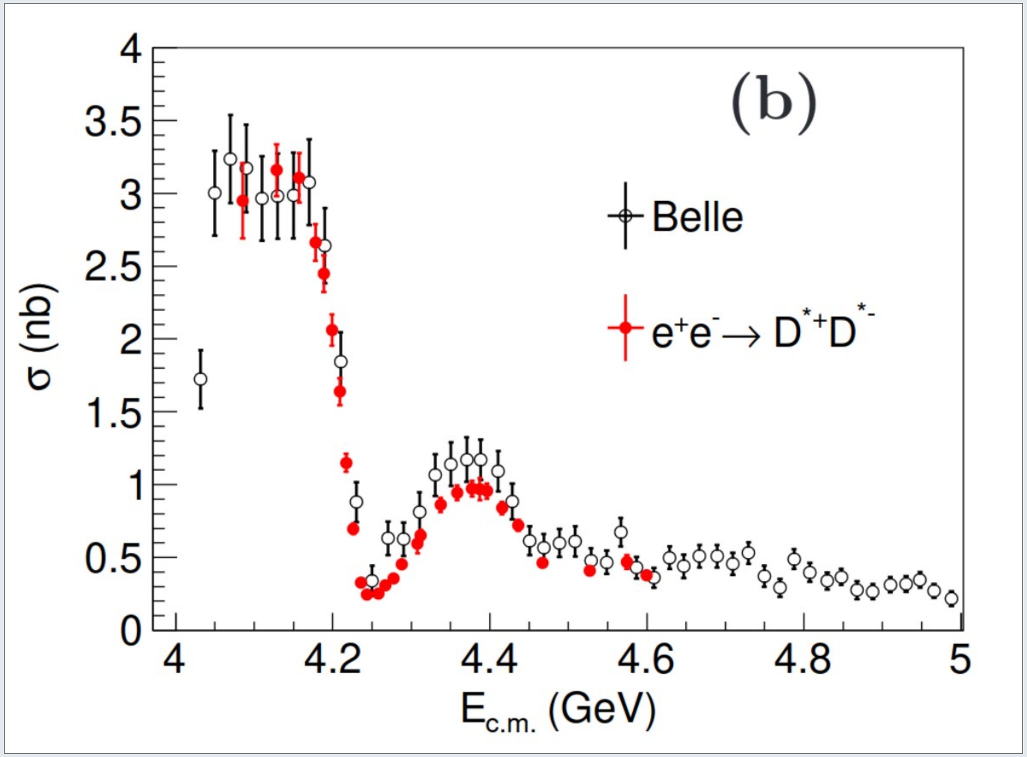
The fit converges… **but**:

1) **Very large** sub-threshold couplings:
   $g(\psi(2S), D_s^*\overline{D}_s^*) \sim 1000 \times g(\psi(2S), D\overline{D})$

2) **Very large** K-matrix widths for ψ(4160) and ψ(4415): 10 – 100 x PDG

3) Experimental **inconsistencies?**

4) **Main issue**: highly correlated parameters



Open questions, wish list

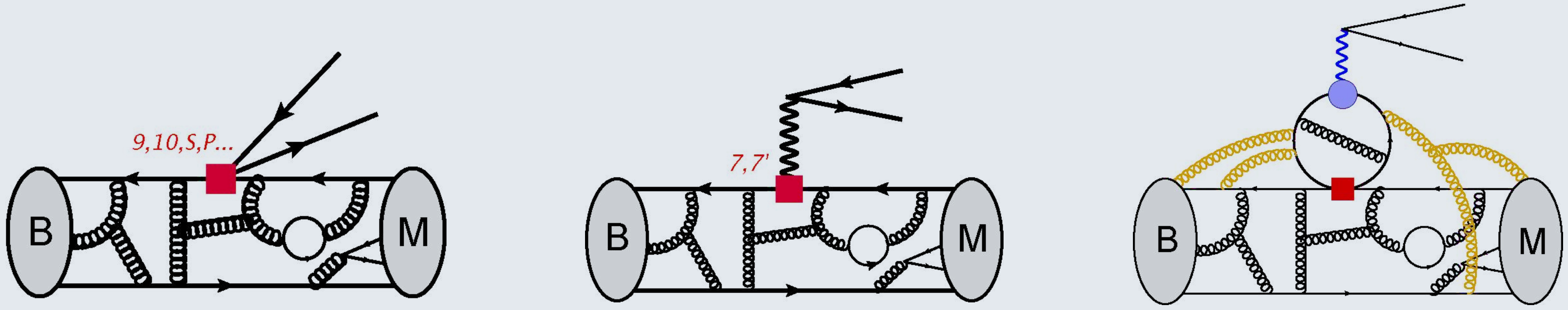
- **Theory:**
  1) Should/How can we use BESIII's **ee → μμ** ?
  2) **Other resonances** seem to be needed, do they have to couple to each channel?
  3) Can **lattice** fix some parameters?
  4) Is it safe to set $\hat{c}_{ij}$ coefficients to **zero**?

- **Fit:**
  1) Can we **decorrelate** the parameters?

- **Experiment:**
  1) **Tagged** analysis $D^0\bar{D}^{*0}$ vs. $D^{*0}\bar{D}^0$
  2) **Angular** analysis of $D_s^*\bar{D}_s^*$
  3) Any measurement of ψ(4230), ψ(4260), ψ(4360), ψ(4660)

Back-up slides

Digression on b→sℓℓ transitions



$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

Non-local form-factors

$$\mathcal{H}_\lambda(q^2) = i\mathcal{P}_\mu^\lambda \int d^4x\, e^{iq\cdot x} \langle \bar{M}_\lambda(k) | T\{\mathcal{J}_{\rm em}^\mu(x), \mathcal{C}_i \mathcal{O}_i(0)\} | \bar{B}(q+k)\rangle$$

Factorization approximation [Kruger & Sehgal `96; Lyon & Zwicky ‘14; Braß, Hiller *et al* ‘16]

$$\mathcal{H}_\lambda^{\rm KS}(q^2) = (C_F \mathcal{C}_1 + \mathcal{C}_2)\, \Pi(q^2)\, \mathcal{F}_\lambda(q^2)$$

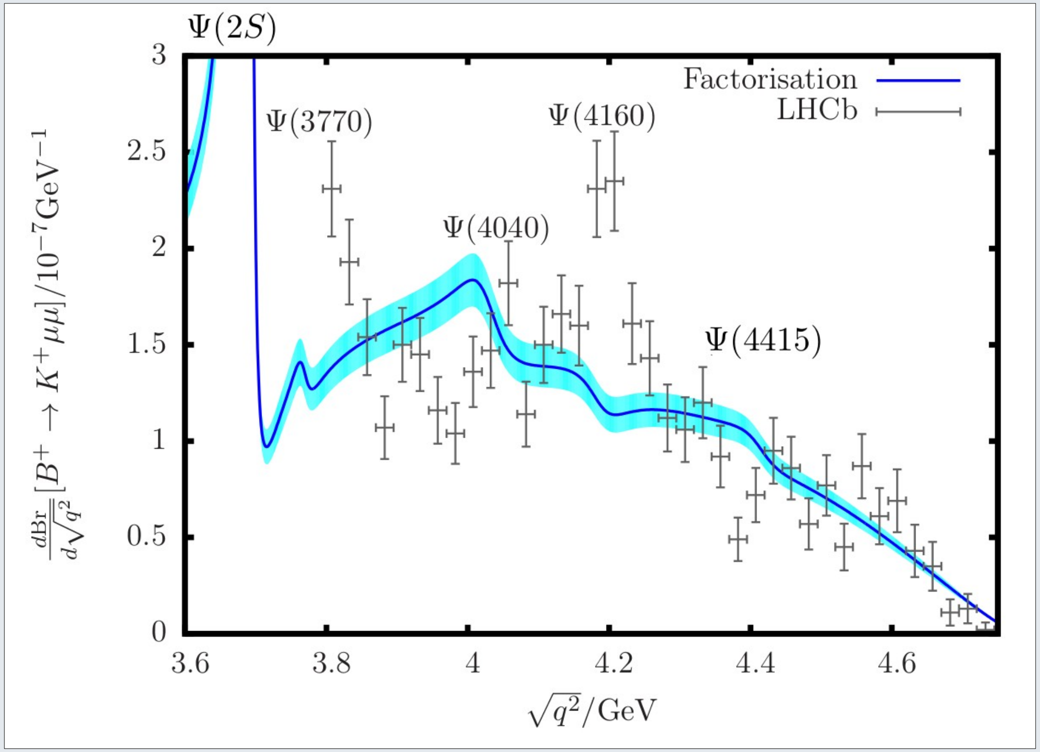
$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu\mu)} \propto \text{Im}\, \Pi(q^2)$$

Back to b→sℓℓ

[Lyon & Zwicky '14, Braß et al. '17]

A posteriori check of the factorization approach

- Requires **additional factors**
- R ratio **cannot correctly reproduce resonances** e.g. ψ(3770) is a **D-wave** resonance so its decay constant vanishes in the non relativistic limit!



Back to b→sℓℓ

Beyond naive factorization, we use a more general approach

$$\operatorname{disc} \mathcal{H}_\lambda^{\rm res}(q^2) \sim \sum_\psi \frac{\mathcal{A}(\psi \to \ell\ell)\, \mathcal{A}(BK^{(*)} \to \psi)}{m_\psi^2 - q^2 + i\Gamma_\psi m_\psi}$$

Fix as many parameters **from data** as possible using:

$$\operatorname{disc} \mathcal{A}(e^+e^- \to D\bar{D}) \propto \sum_\psi \frac{\mathcal{A}(\psi \to D\bar{D})\, \mathcal{A}(\psi \to e^+e^-)}{m_\psi^2 - q^2 + i\Gamma_\psi m_\psi}$$

(BES, BaBar, Belle)

$$\operatorname{disc} \mathcal{A}(B \to K^{(*)} D\bar{D}) \propto \sum_\psi \frac{\mathcal{A}(\psi \to D\bar{D})\, \mathcal{A}(BK^{(*)} \to \psi)}{m_\psi^2 - q^2 + i\Gamma_\psi m_\psi}$$

(LHCb, BaBar, Belle)

Global ee→$c\bar{c}$ fit in EOS

- The fit we perform is:
  - **global** = we used all experimental data
  - **extendable** = significance of new resonances can be studied
  - Implemented in **EOS**
- Main challenges: **Fit performances**, **estimation of uncertainties**

EOS is a software for a variety of applications in flavour physics. It is written in C++, but provides an interface to Python.

→ *EOS paper: 2111.15428*

https://eos.github.io/

Features of the implementation

- K-matrix is implemented in **EOS**
  - **Fast numerical evaluation**
    - Written in C++
    - Efficiency due to caching of intermediate results
  - **Versatile**
    - Not limited to $ee \to c\bar{c}$
    - Adjustable number of channels/resonances
    - Polymorphic object for the channels (adjustable phase space factors, centrifugal barrier factors...)
- https://github.com/eos/eos
- New features/observables can be implemented!